

# Применение сеточно-характеристического метода для решения задач распространения динамических волновых возмущений на высокопроизводительных вычислительных системах

Петров Игорь Борисович, член-корр. РАН, д.ф.-м.н, МФТИ  
Хохлов Николай Игоревич, к.ф.-м.н, МФТИ

Открытая конференция ИСП РАН им. В.П. Иванникова  
Москва, 5-6 декабря 2019 г.

# Numerical Methods and Models

- Media models:
  - Linear elasticity, viscoelastic
  - Linear acoustic
- Numerical methods:
  - Grid-characteristic
  - Galerkin
  - FDTD
  - TVD, WENO finite-volumes
- Features:
  - Explicit cracks setting
  - Explicit layers borders setting
  - High-order numerical schemes, monotonicity
  - HPCs oriented

# Mathematical model

Relation between velocity and deformation

$$\left\{ \begin{array}{ll} \rho \dot{\vec{v}} = \nabla \cdot \mathbf{T} & \text{Motion equation} \\ \dot{\mathbf{T}} = \lambda(\nabla \cdot \vec{v})\mathbf{I} + \mu(\nabla \otimes \vec{v} + \vec{v} \otimes \nabla) & \text{Hooke's law} \end{array} \right.$$

$\rho$  – density

$\lambda, \mu$  – Lamé elastic parameters

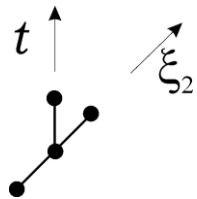
$\vec{v}$  – velocity

$\mathbf{T}$  – stress tensor

# Grid-characteristic method

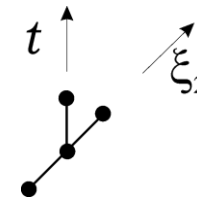
## Split directions

$$\vec{u} = (v_x, v_y, \sigma_{xx}, \sigma_{xy}, \sigma_{yy})$$



$$\frac{\partial \vec{u}}{\partial t} + \mathbf{A}_1 \frac{\partial \vec{u}}{\partial \xi_1} = 0$$

$$\vec{u}' = \vec{u}^n - \tau \mathbf{A}_1 \Delta_1 \vec{u}^n$$



$$\frac{\partial \vec{u}}{\partial t} + \mathbf{A}_2 \frac{\partial \vec{u}}{\partial \xi_2} = 0$$

$$\vec{u}^{n+1} = \vec{u}' - \tau \mathbf{A}_2 \Delta_2 \vec{u}'$$

$$\vec{u}^{n+1} = \vec{u}^n - \tau (\mathbf{A}_1 \Delta_1 + \mathbf{A}_2 \Delta_2) \vec{u}^n + O(\tau^2)$$

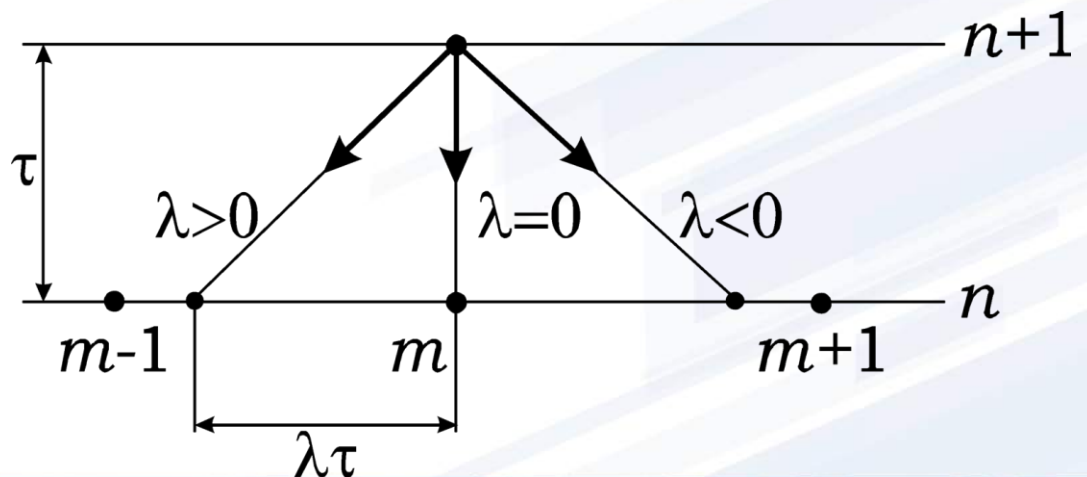
## Hyperbolic problem

$$\mathbf{A} = \Omega^{-1} \mathbf{\Lambda} \Omega$$

$$\frac{\partial \vec{u}}{\partial t} + \Omega^{-1} \mathbf{\Lambda} \Omega \frac{\partial \vec{u}}{\partial \xi} = 0$$

$$\frac{\partial \vec{v}}{\partial t} + \mathbf{\Lambda} \frac{\partial \vec{v}}{\partial \xi} = 0 \quad (\vec{v} \equiv \Omega \vec{u})$$

$$v^{n+1}(\xi) = v^n(\xi - \lambda \tau)$$



# Research Software

- **Structural mesh**

- Grid-characteristic, finite-volume, FDTD methods 1-4 order of accuracy
- Block structural meshes, curved meshes
- Large mesh sizes (over 1 billion nodes)
- Parallelized to MPI / OpenMP / CUDA / OpenCL

- **Non-structural grids**

- The discontinuous Galerkin method of 1-8 order of accuracy
- Adaptive Triangular and Tetrahedral Mesh
- Settlement grids more than 100 million nodes
- Parallelized to MPI / OpenMP

- **Features:**

- VTK-based visualization (ParaView, VisIt, Mayavi)
- Seismogram format - segy
- Elastic / acoustic media, explicit separation of contacts and heterogeneities

## Вычислительный модуль

Блочные структурные сетки,  
иерархические сетки  
2D/3D

Неструктурные сетки,  
адаптивные сетки  
2D/3D

## Препроцессинг

Встроенная система  
конфигурационных  
файлов

Открытые CAD  
пакеты,  
RSF, segy

## Постпроцессинг

На основе  
VTK, segy

# Research Software Features

- Linux console intensive usage
- MPI and OpenMP technologies utilization, remote access to hardware
- No external dependencies, pure C++
- Oriented on C++ 98 (currently porting to C++11) to eliminate compile problems on old hardware

# GPU parallelization

- CPU
  - Compilers: icc
  - Compiler Options :
    - -mavx
    - -fopenmp (auto vectorization)
    - -O2
- GPU
  - Compilers: nvcc, gcc
  - Compiler Options:
    - -O2
    - -use\_fast\_math

# CPU properties: Intel Xeon E5-2697 2.7 GHz

## GPU properties:

GPU	CUDA cores (streaming processors)	Clock rate, MHz	GFLOPS - single precision	SP:DP	GFLOPS - double precision
GeForce GT 640	384	900	691	24	29
GeForce GTX 480	480	1401	1345	8	168
GeForce GTX 680	1536	1006	3090	24	129
GeForce GTX 760	1152	980	2258	24	94
GeForce GTX 780	2304	863	3977	24	166
GeForce GTX 780 Ti	2880	876	5046	24	210
GeForce GTX 980	2048	1126	4612	32	144
Tesla M2070	448	1150	1030	2	515
Tesla K40m	2880	745	4291	3	1430
Tesla K80	2496	562	2806	1.5	1870
Radeon HD 7950	1792	800	2867	4	717
Radeon R9 290	2560	947	4849	8	606



# Test program

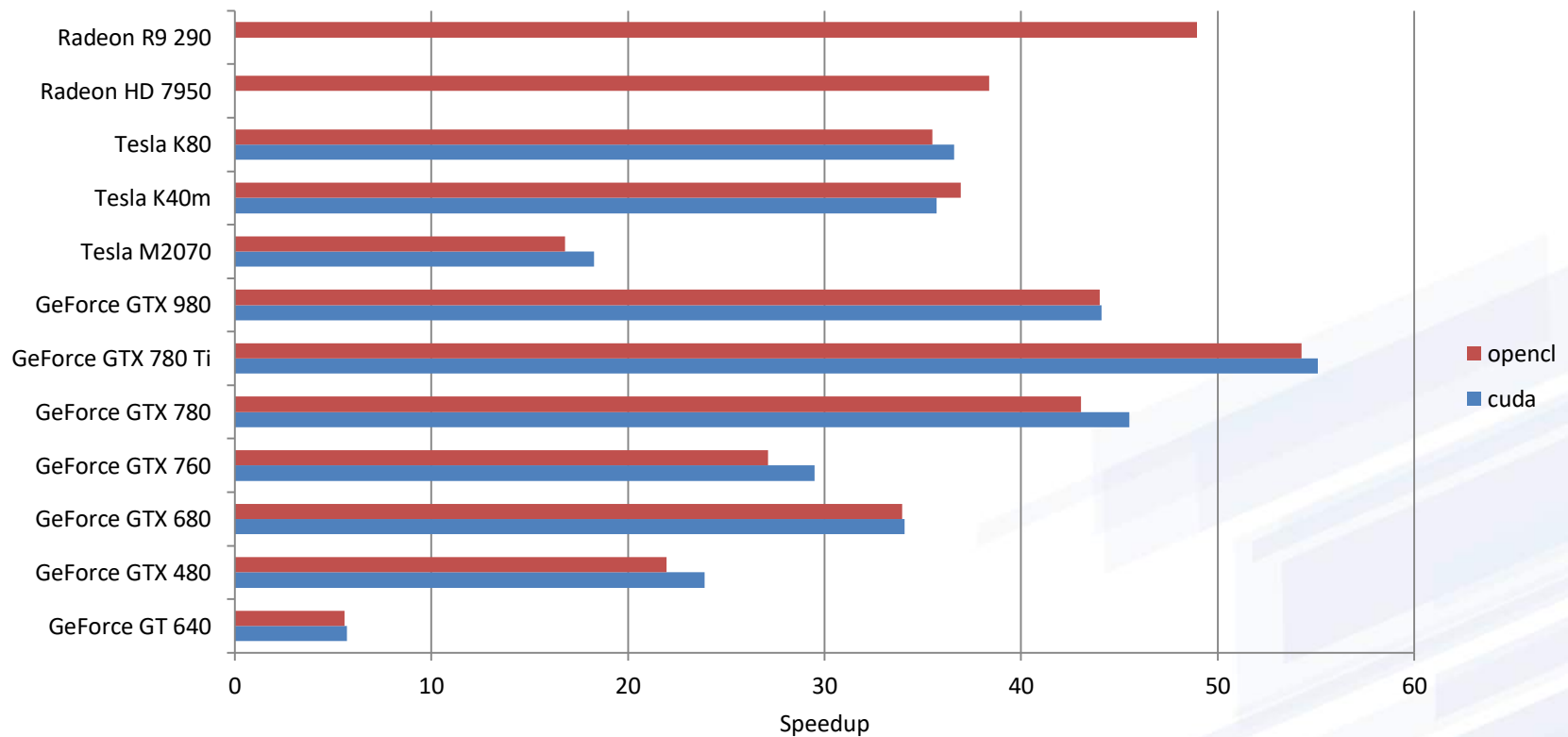
- Grid size: 4096x4096
- Time steps: 6500
- Data type: float, double
- Grid node: 5 float (double)
- Occupied memory:
  - 320 MB (float)
  - 640 MB (double)

# CPU version

- Single-precision and double-precision
- 190 FLOPS to recalculate one node in grid
- Program consumes 18.8 TFLOPS
- Single-thread, single CPU core
- AVX instructions – vectorization

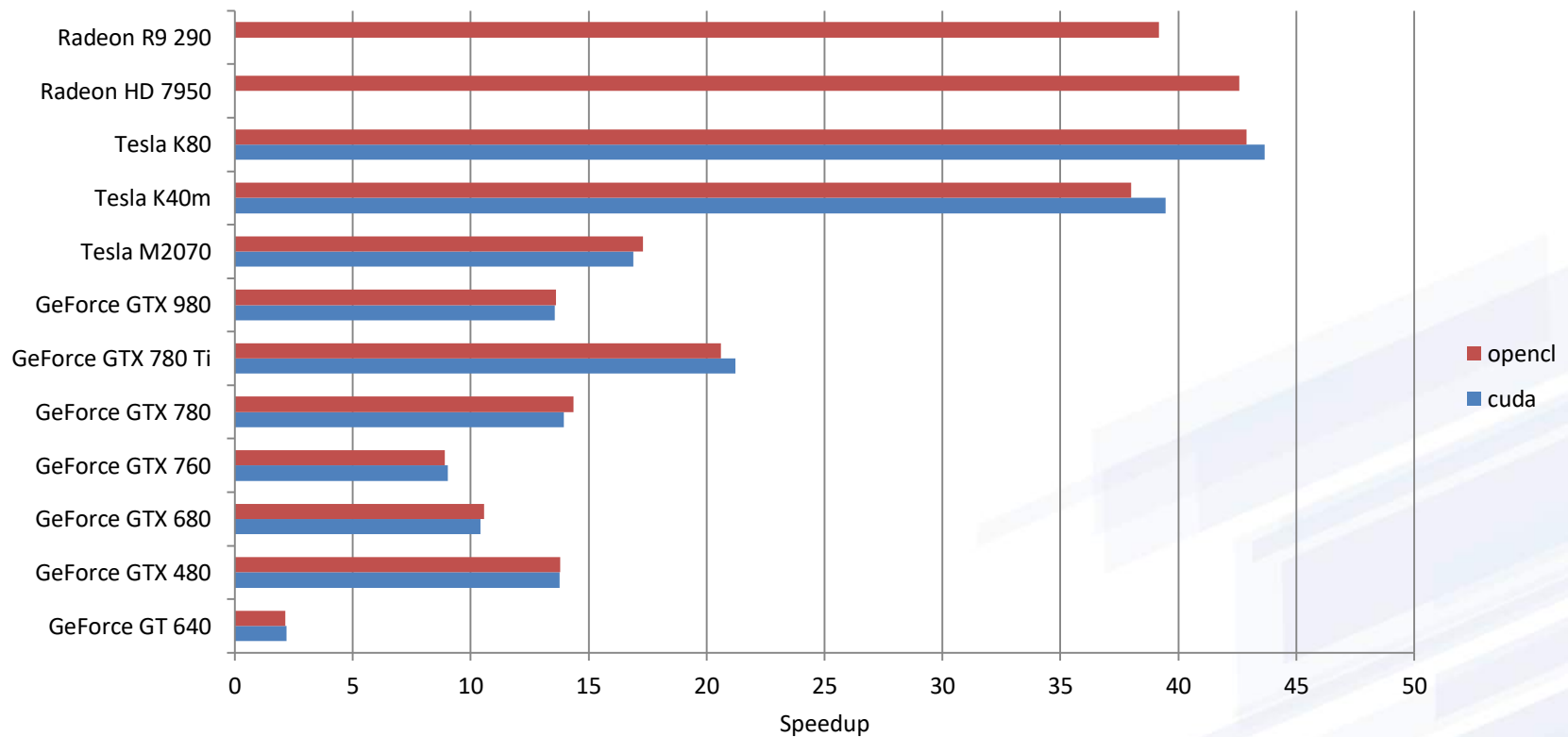
# Speedup of GPU implementation compared to CPU

compare with cpu Intel Xeon E5-2697 - float + fast math



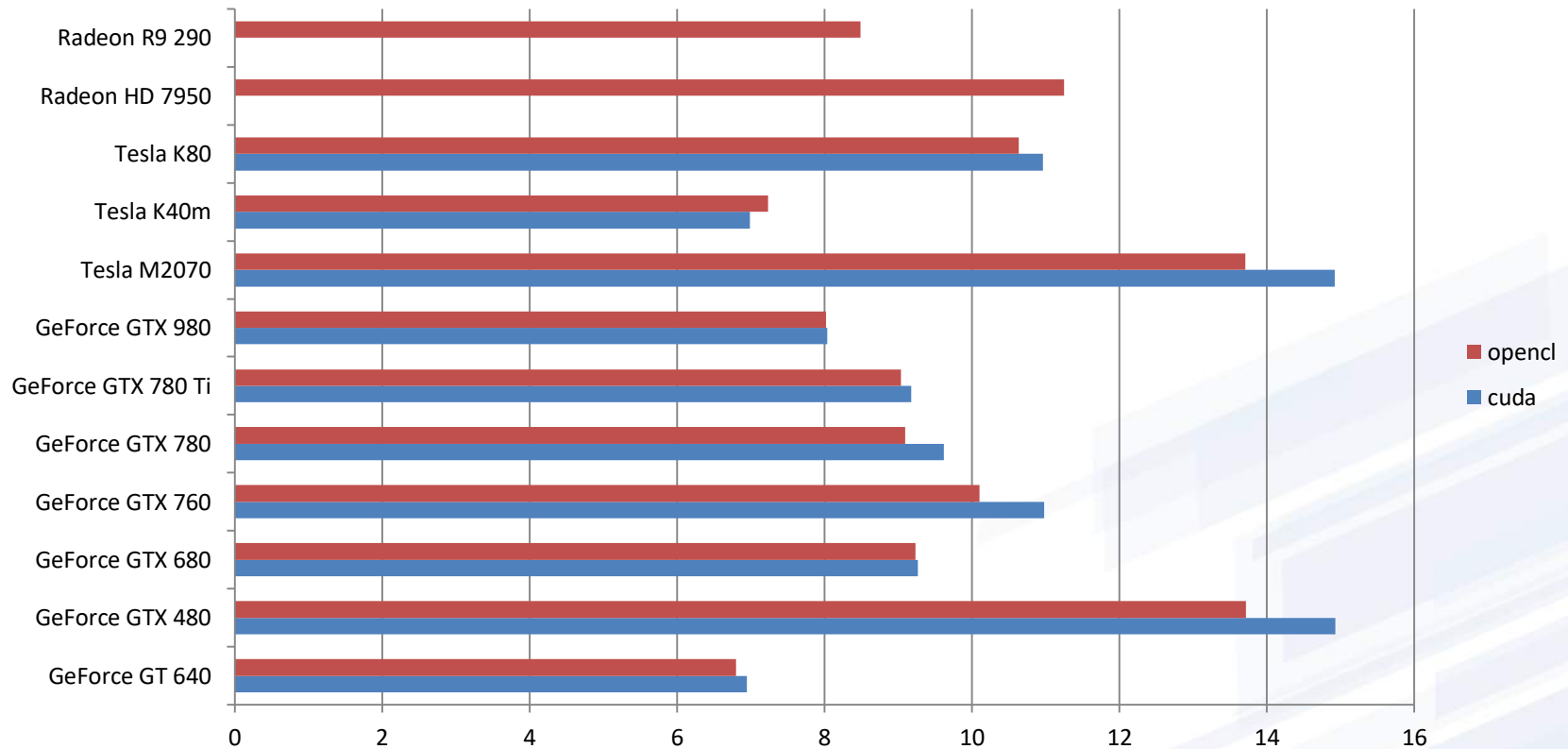
# Speedup of GPU implementation compared to CPU

compare with cpu Intel Xeon E5-2697 - double



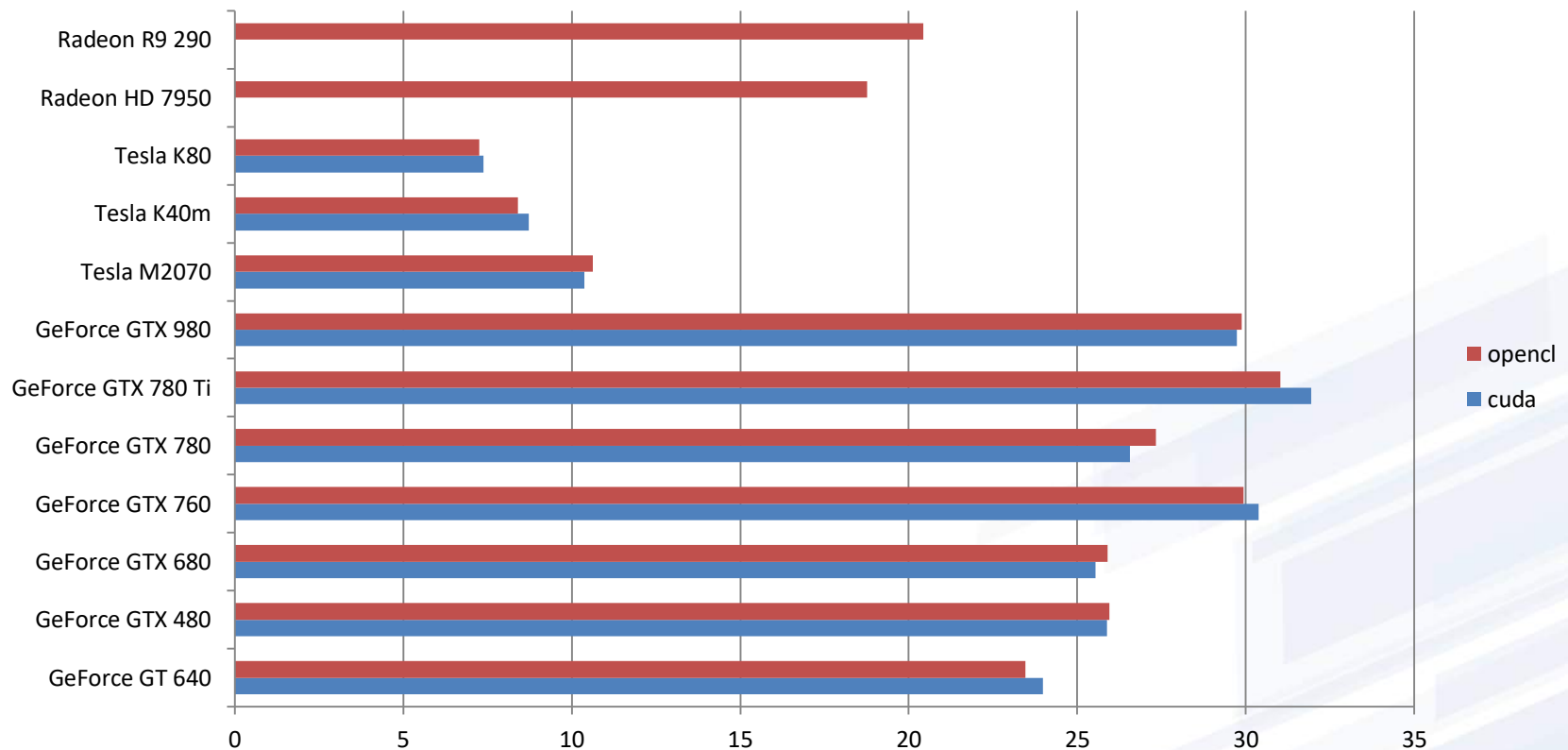
# Percentage of peak performance

Percentage of peak performance - float + fast math



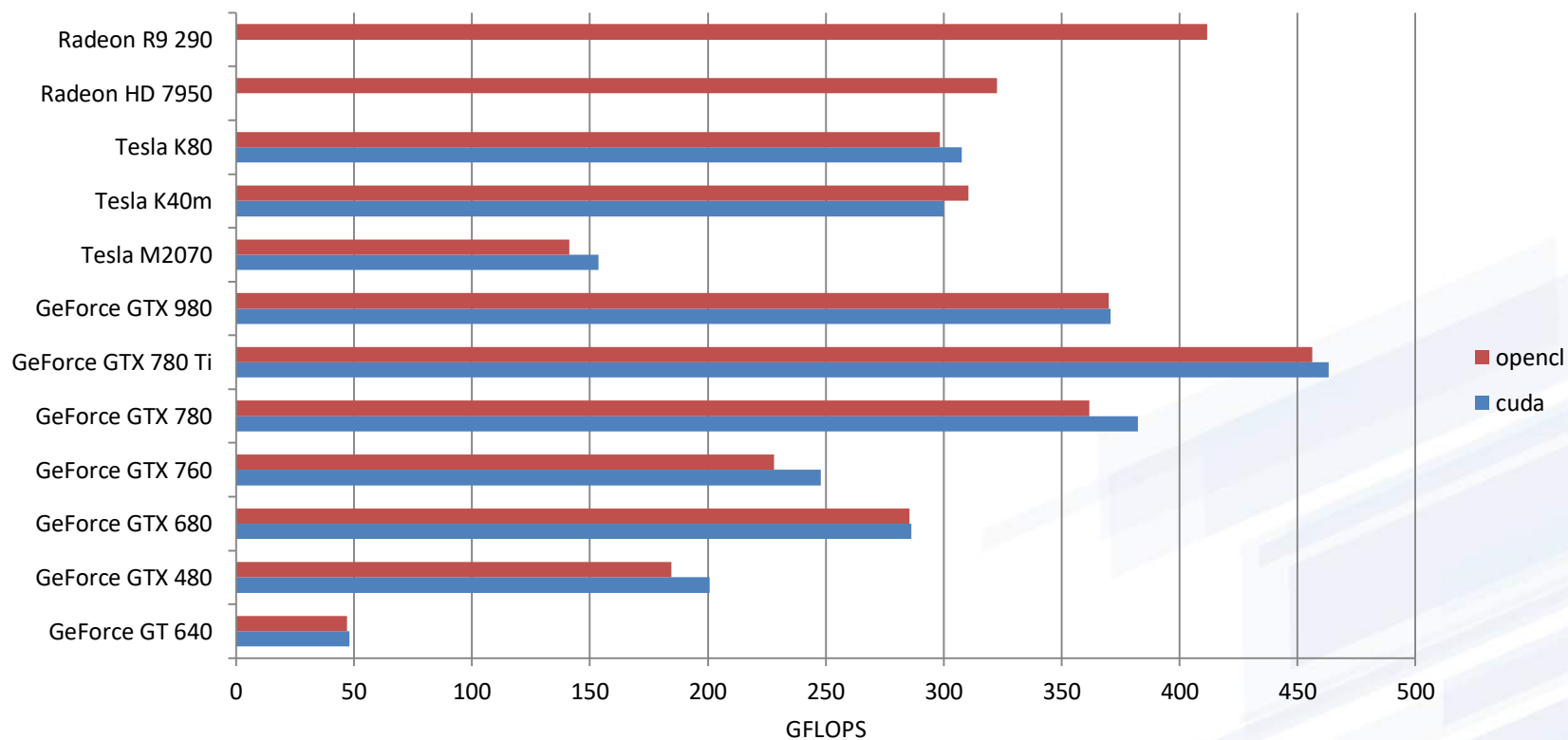
# Percentage of peak performance

Percentage of peak performance - double

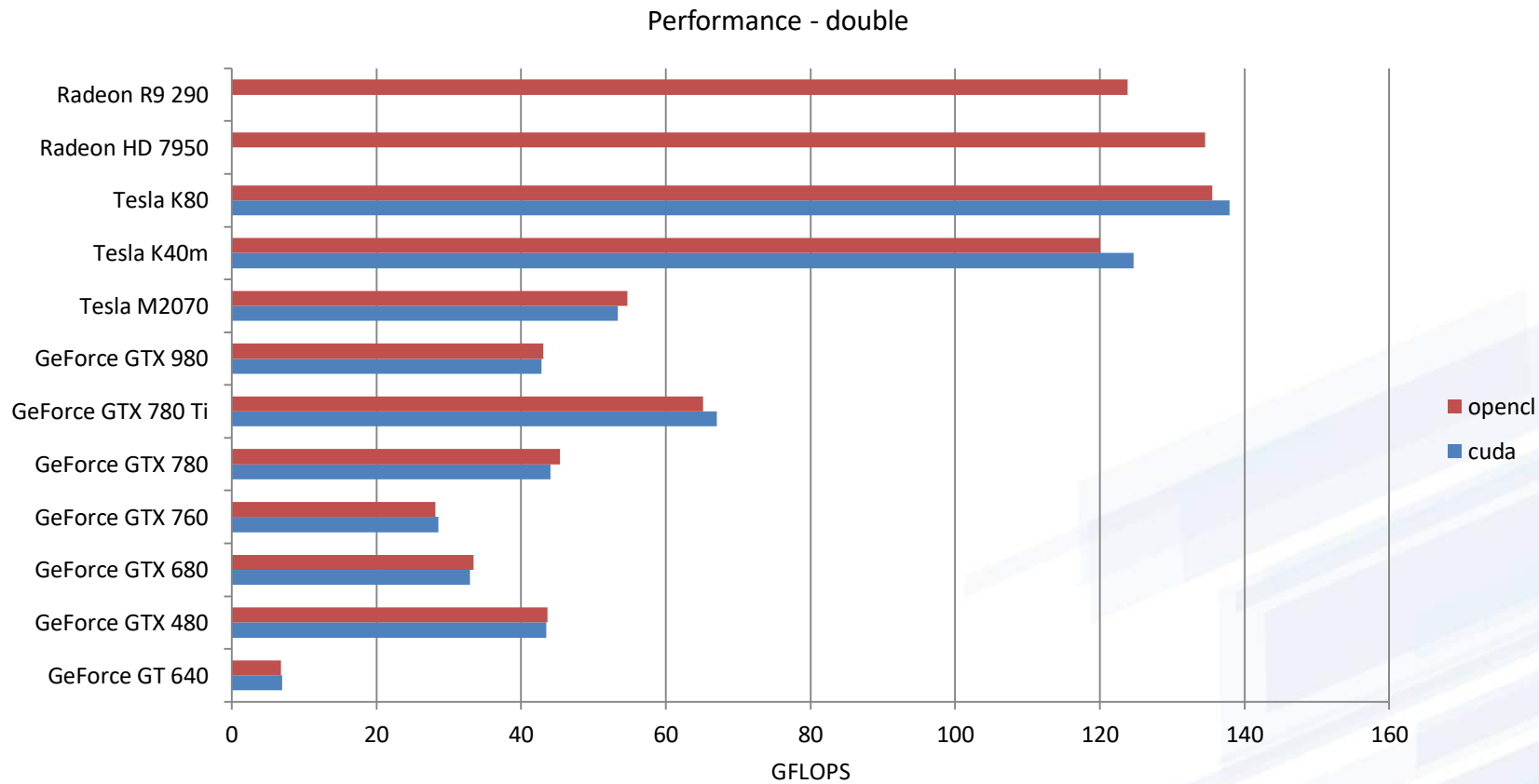


# Performance

Performance - float + fast math



# Performance



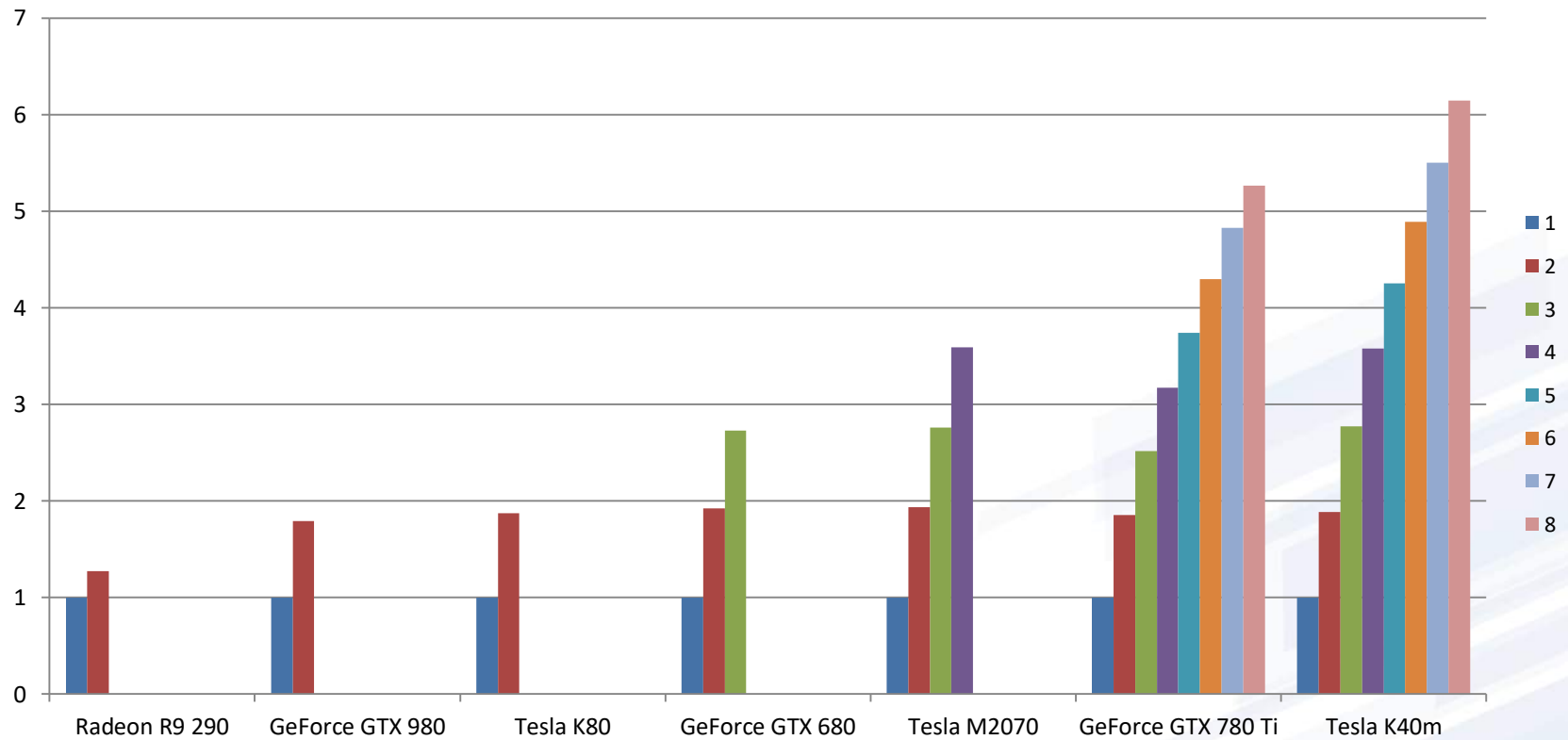


# GPU parallelization

- Multiple GPUs
- Divide grid along axis Y
- Data exchanges between GPUs by adjacent grid nodes
- GPUDirect (only in CUDA) – exchange data by PCI Express bypassing CPU

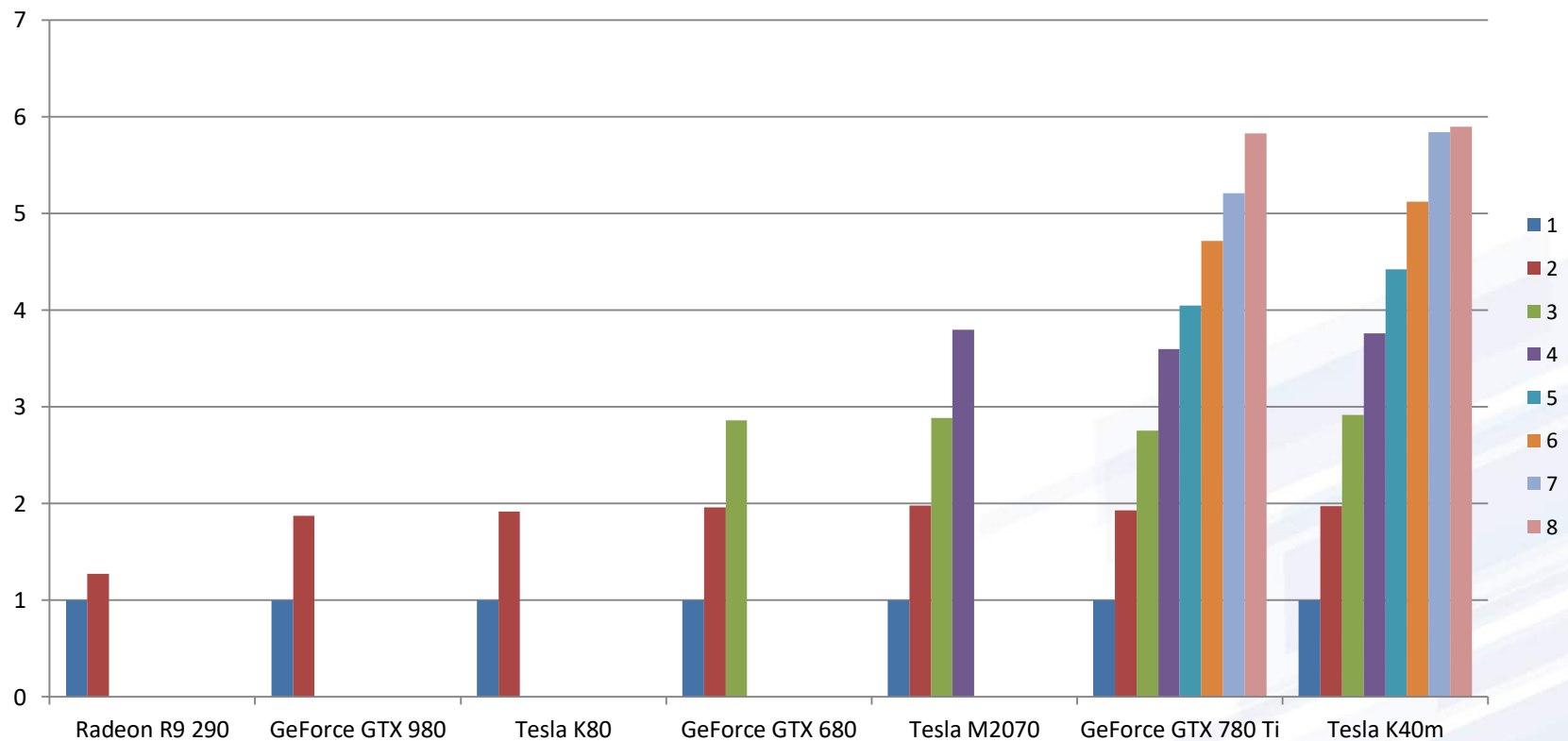
# Speedup (number of GPUs)

## Speedup, float

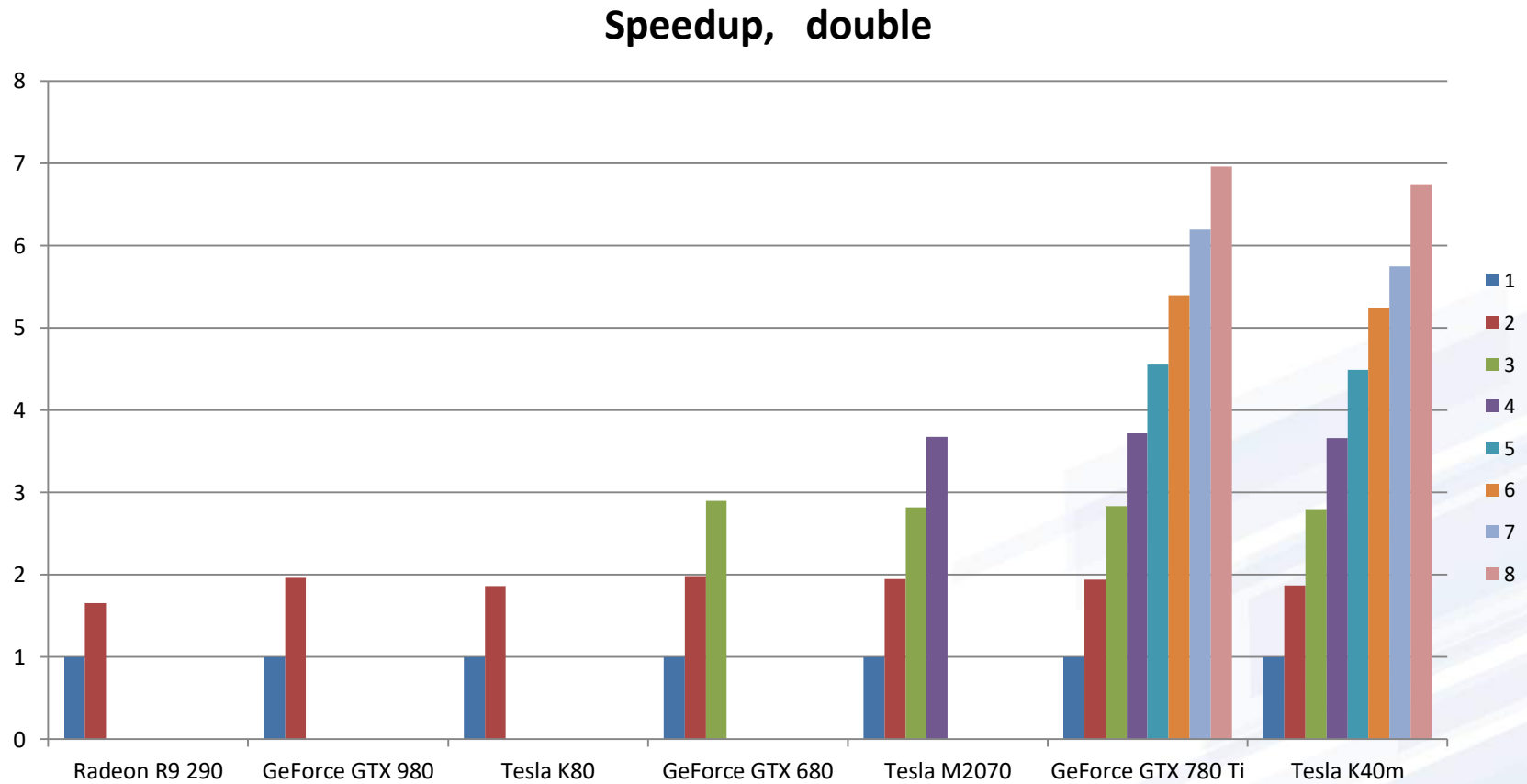


# GPUDirect (except Radeon R9 290)

## GPUDirect, float

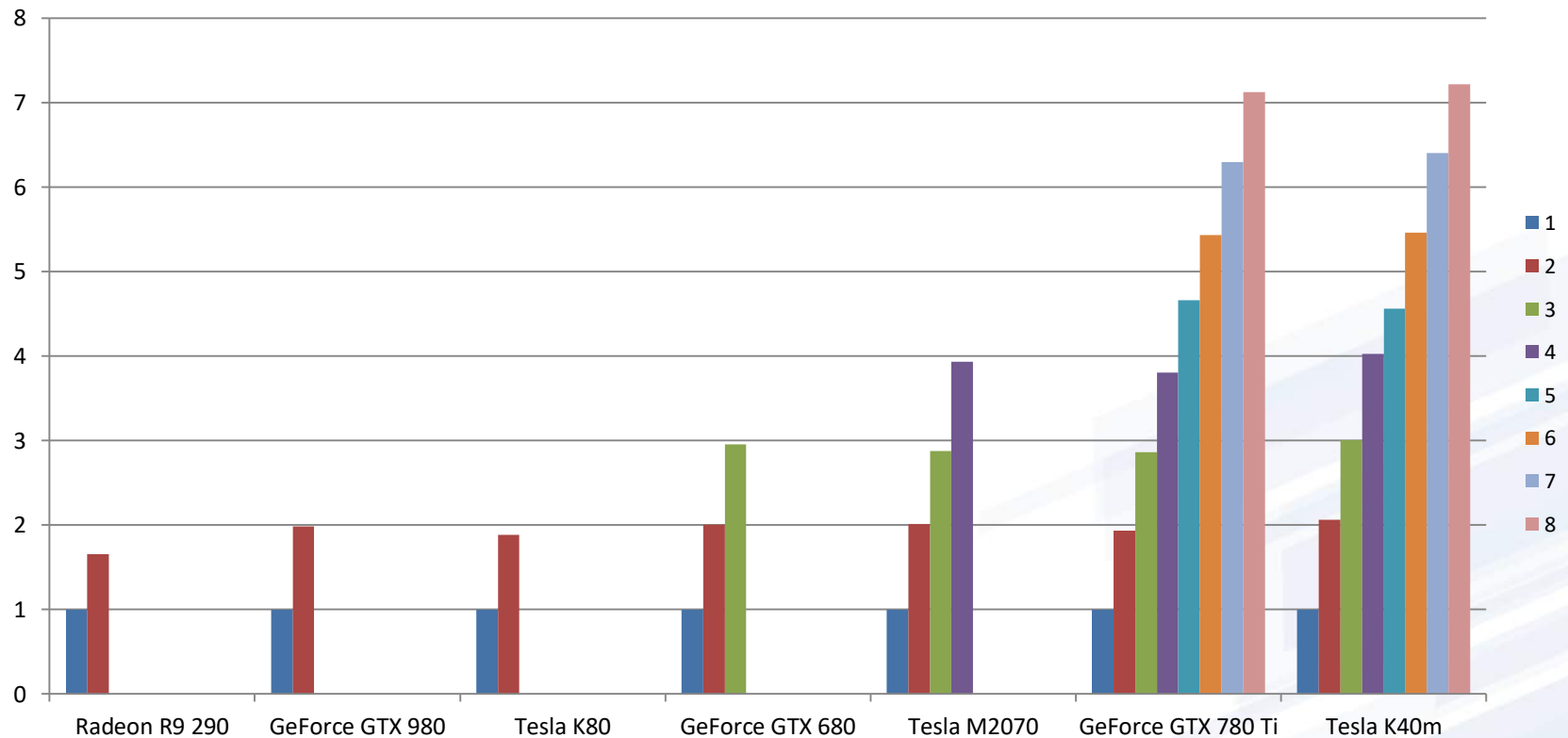


# Speedup (number of GPUs)



# GPUDirect (except Radeon R9 290)

## GPUDirect, double



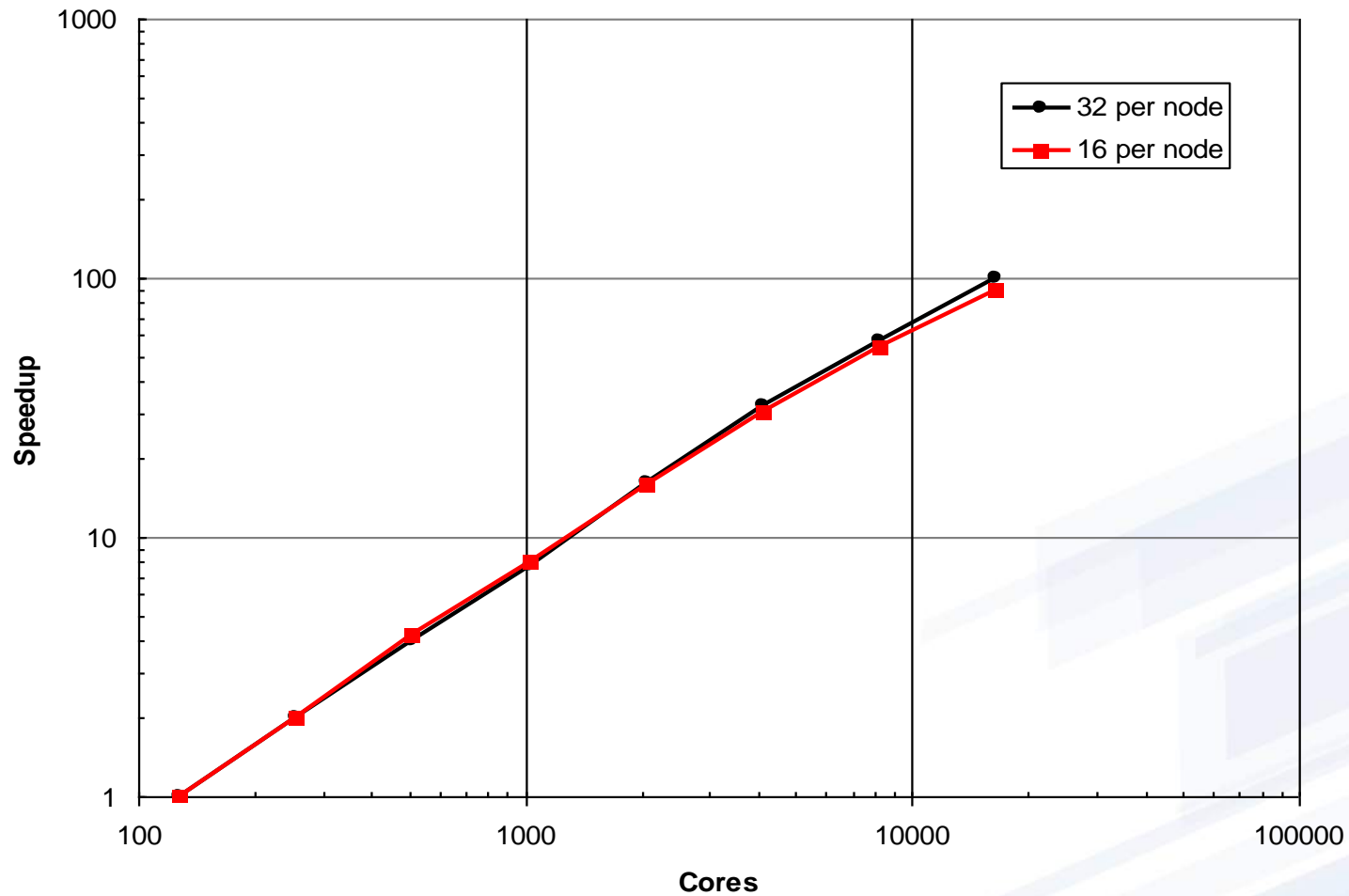
# Conclusion

- Speedup (single GPU compared with CPU):
  - Single-precision – up to **55** times (**GeForce GTX 780 Ti**)
  - Double-precision – up to **44** times (**Tesla K80**)
- Performance (single GPU):
  - Single-precision – up to **460** GFLOPS (**GeForce GTX 780 Ti**)
  - Double-precision - up to **138** GFLOPS (**Tesla K80**)
- Speedup (multiple GPU compared with single GPU):
  - Single-precision – up to **6.1** times (**Tesla K40m**)
  - Double-precision – up to **7.1** times (**GeForce GTX 780 Ti**)
- Increase in speedup with GPUDirect
  - Single-precision - **10%** on 8 **GeForce GTX 780 Ti**
  - Double-precision – **2.4%** on 8 **GeForce GTX 780 Ti**

# MPI: cluster HECToR

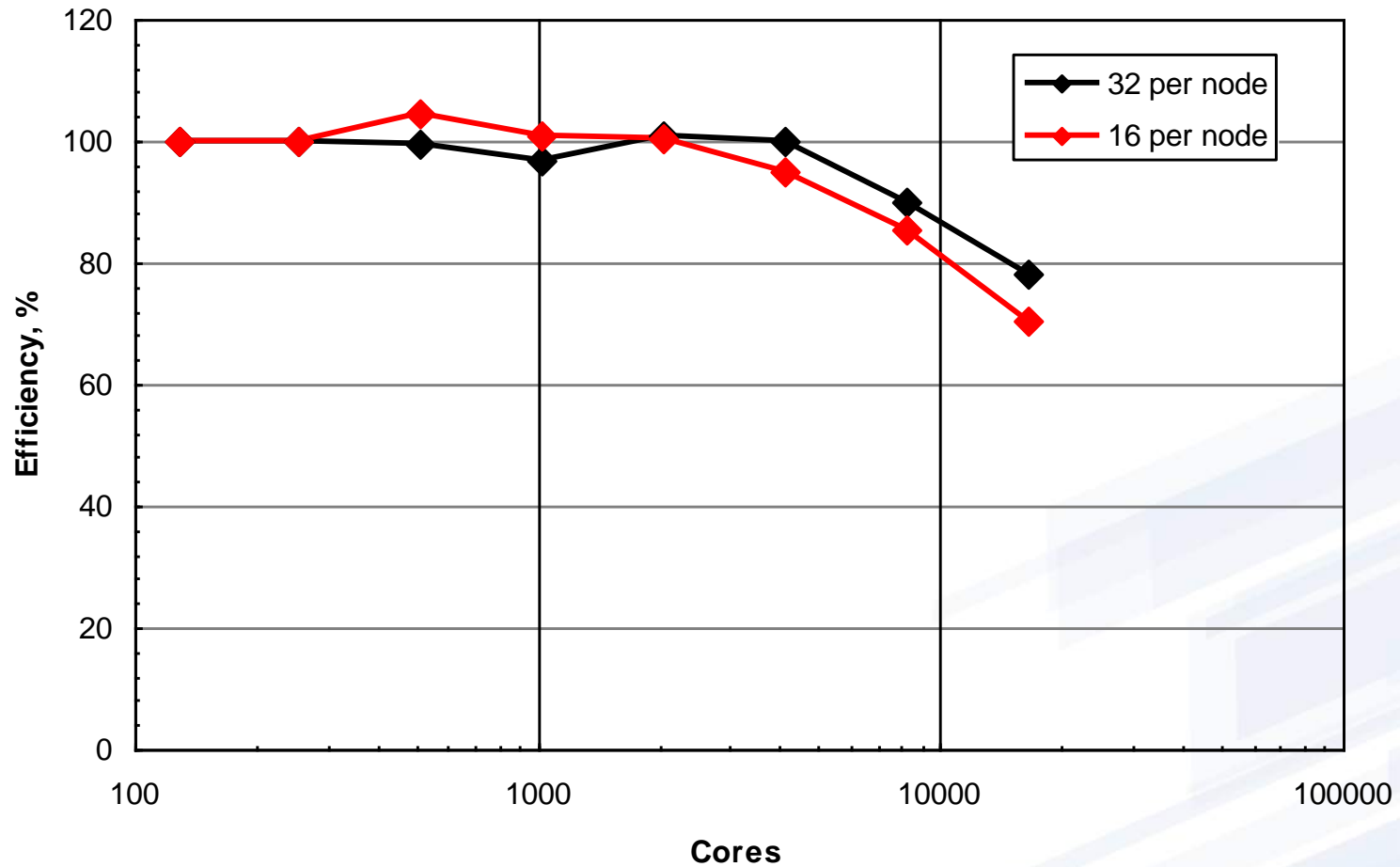
- ~90 000 cores
- 1 GB memory per core
- 32 cores per node
- Test on a grid 1000x1000x1000 - 1 billion nodes
- The number of cores from 128 to 16 thousand
- The number of threads per node - 16 and 32
- Measurement only of the general operating time of the program
- Acceleration - 100 and 90 times
- Efficiency - 78 and 70%

# MPI: strong scalability





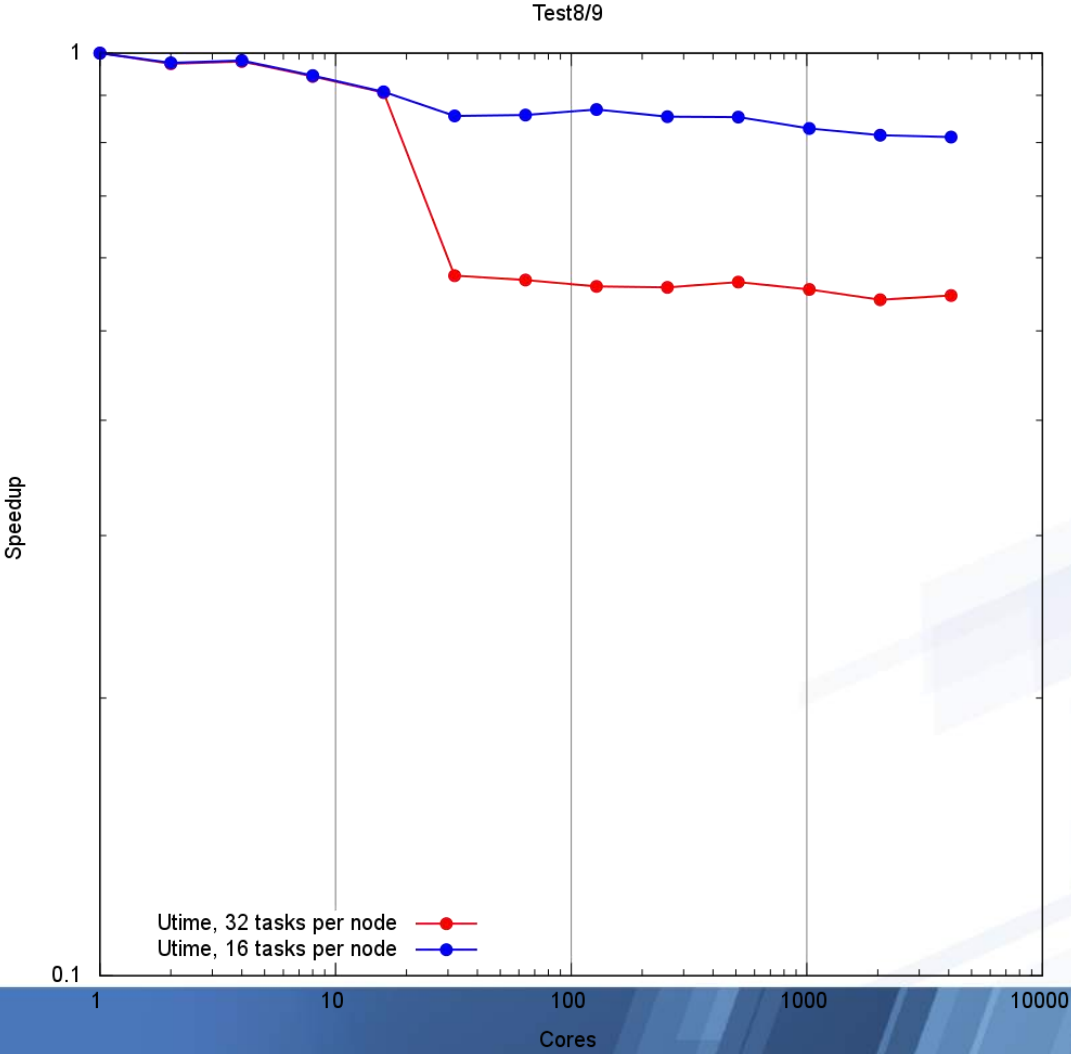
# MPI: efficiency



## Weak scalability

- The number of grid nodes per core - 16 million
- Maximum grid size - 62 billion nodes
- Good performance
- Difference in calculations for 16 and 32 threads per node
- Acceleration - 0.8 and 0.55 respectively

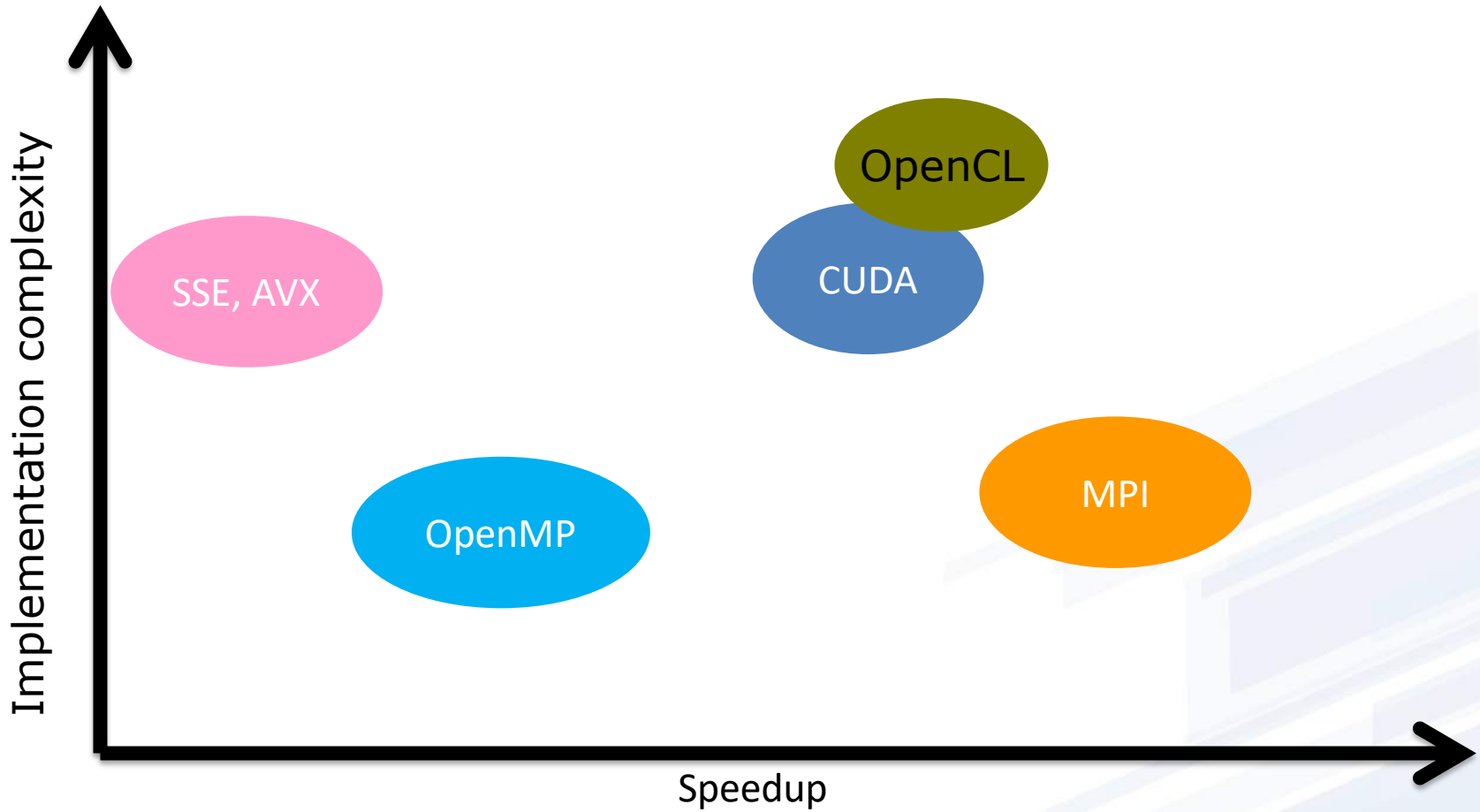
# Weak scalability



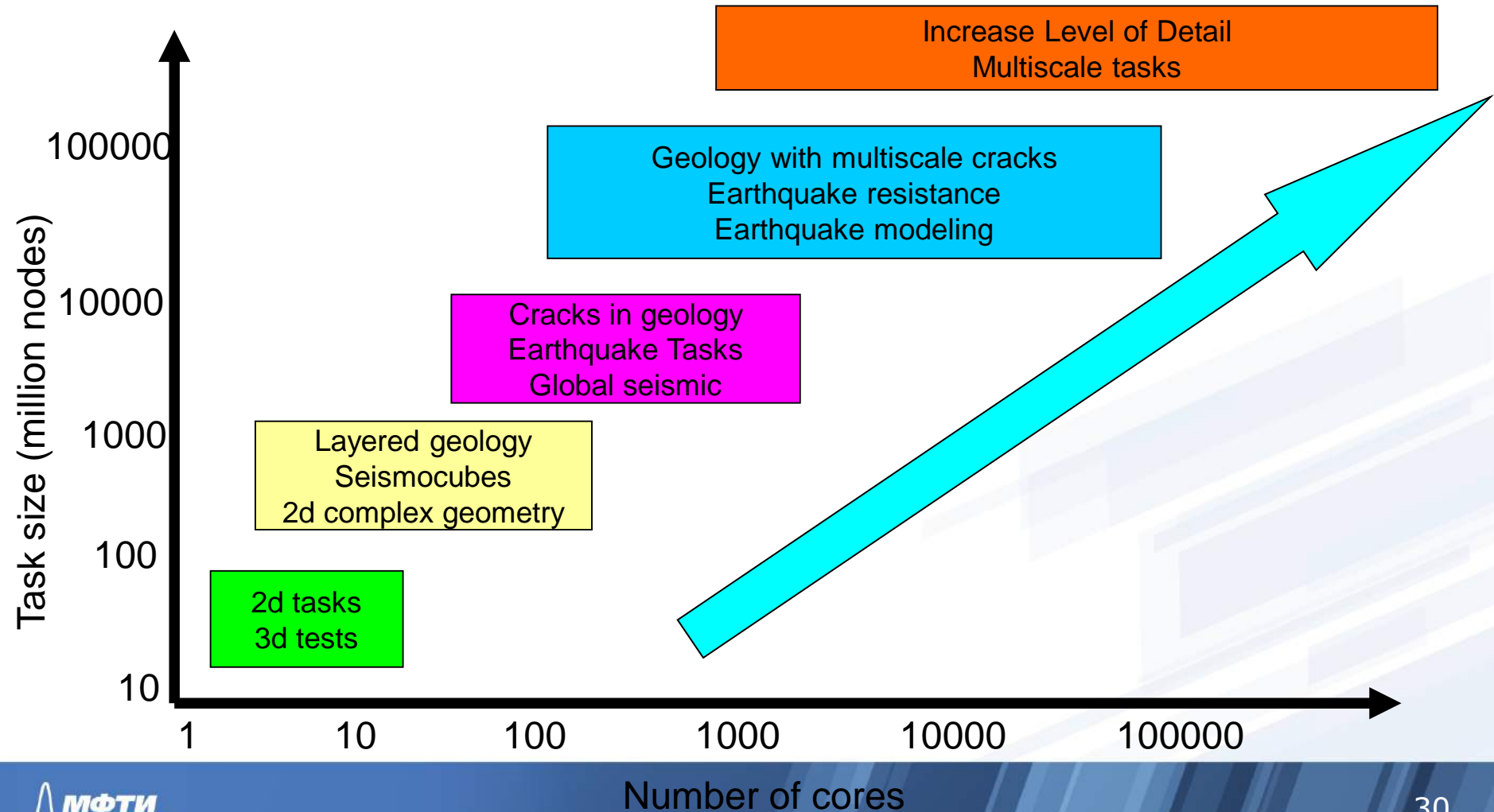
# OpenMP parallelization

- The highest performance of the serial version is 22% of the peak for float and 17% for double with optimization and using avx instructions
- Acceleration on AMD Opteron 6272 - 37 times on 64 cores
- Acceleration on AMD Opteron 8431 - 25 times on 48 cores
- Acceleration on Intel Xeon E5-2697 - 17 times on 24 cores

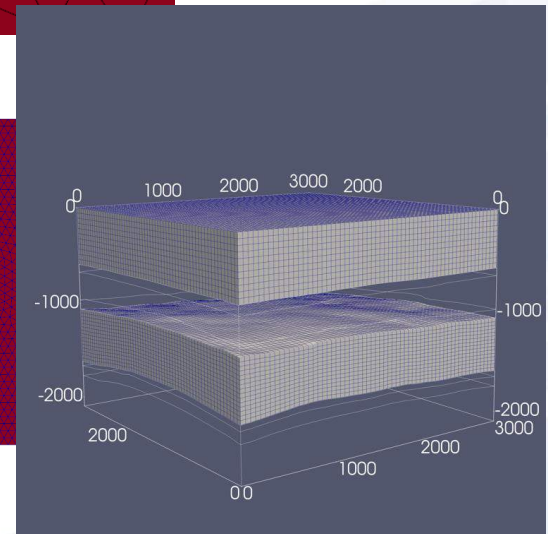
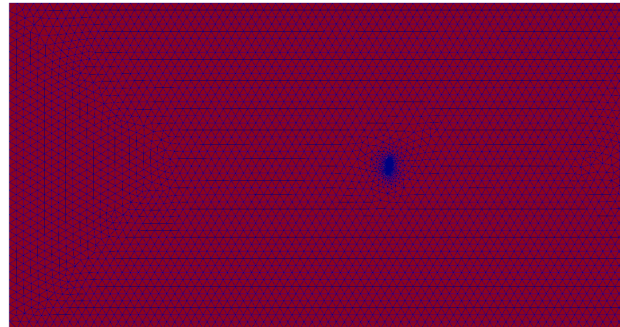
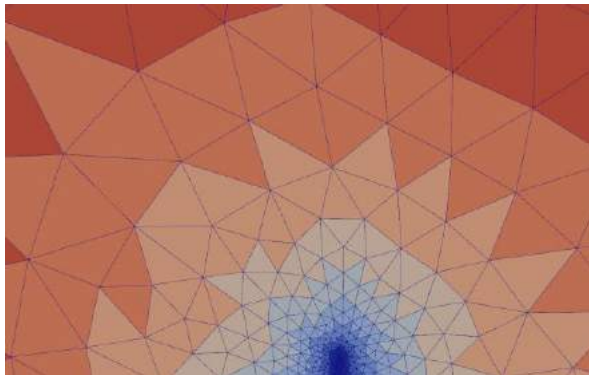
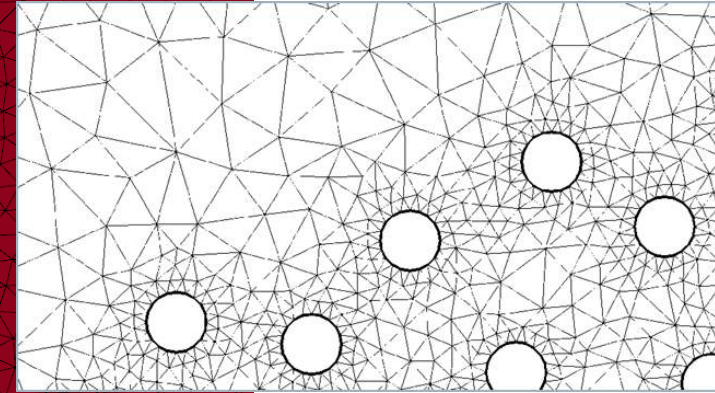
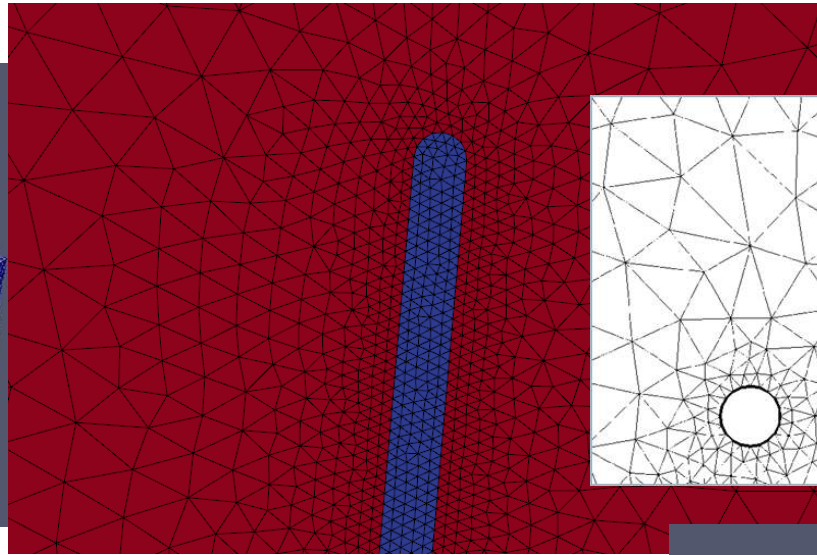
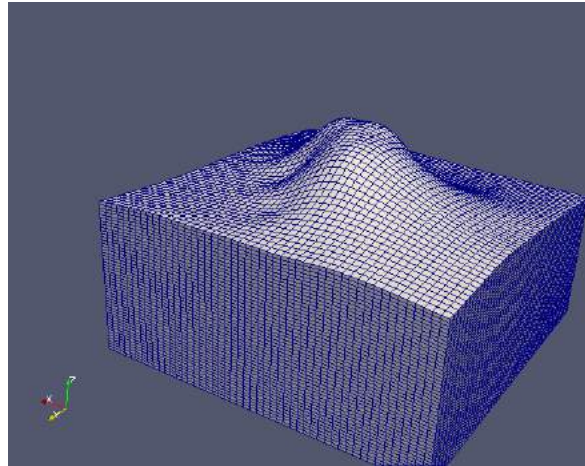
# Implementation complexity



# Task size



# Examples of Meshes

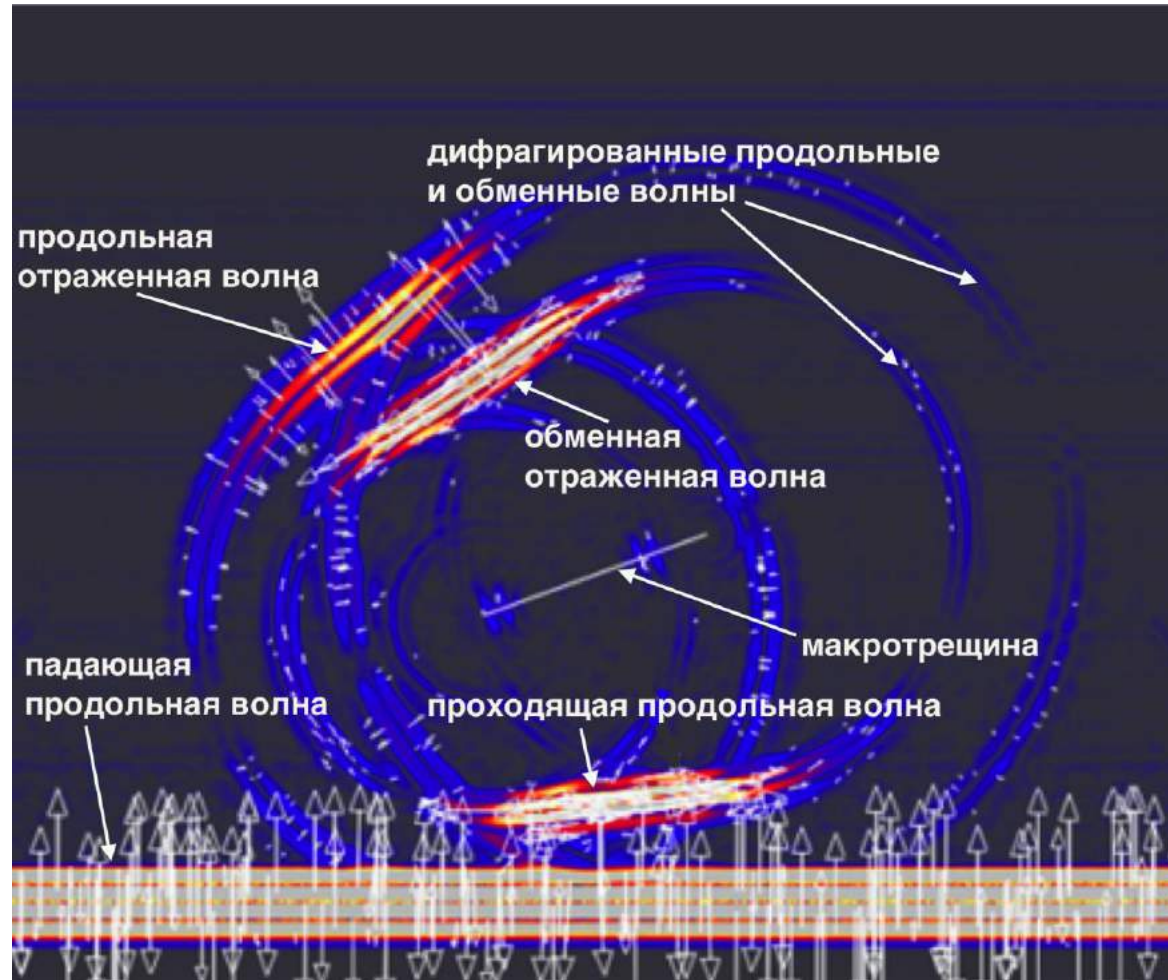


# Seismic modeling



# Geological Cracks Simulation

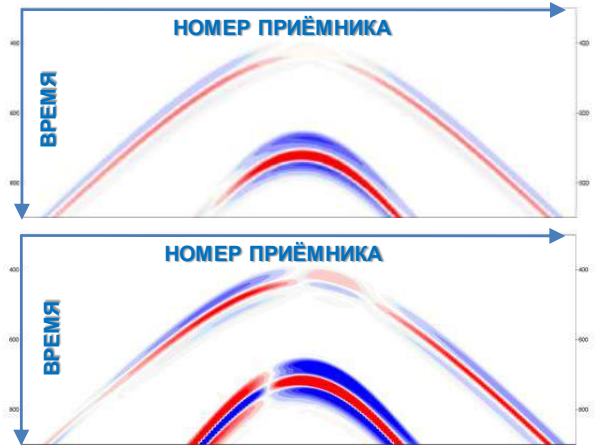
- Explicit setting of medium internal structure
- Crack-crack interaction is taken into account explicitly



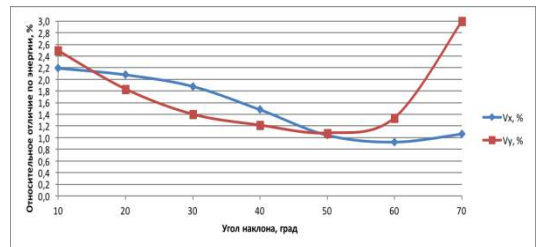
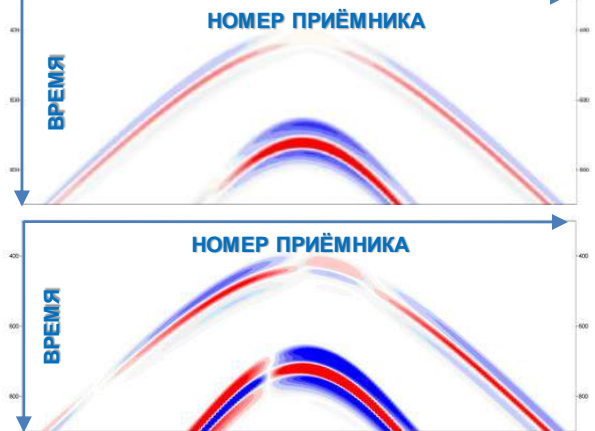
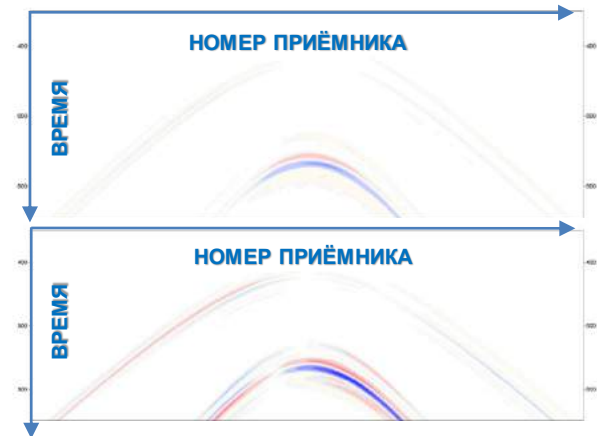
*A set of publications by Petrov, Leviant, etc.*

# Fracture model

## РАСКРЫТИЕ 1:1000



## РАЗНИЦА СЕЙСМОГРАММ

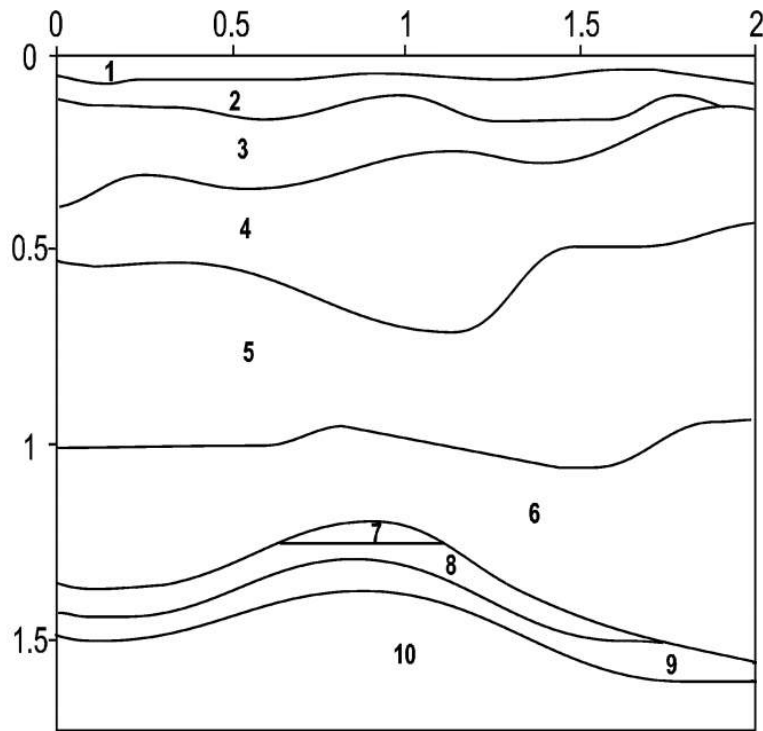


**АНАЛИЗ, РАЗНИЦА <3%**

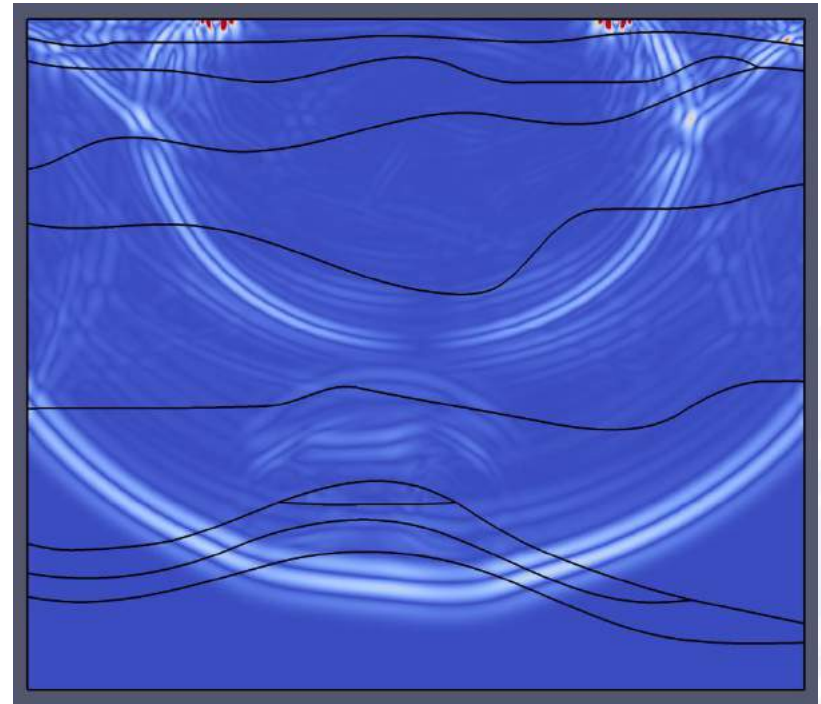
**МОДЕЛЬ ТРЕЩИН БТТ**

**МОЖНО ИСПОЛЬЗОВАТЬ  
МОДЕЛЬ БТТ**

# Example problem

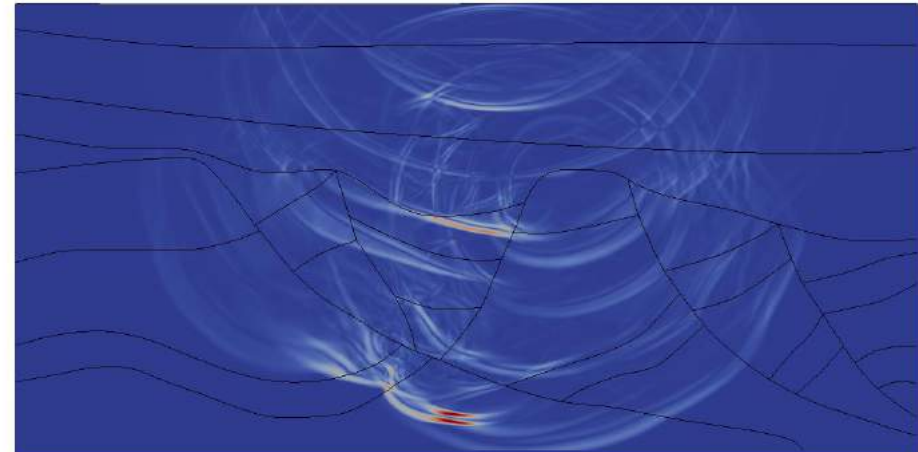
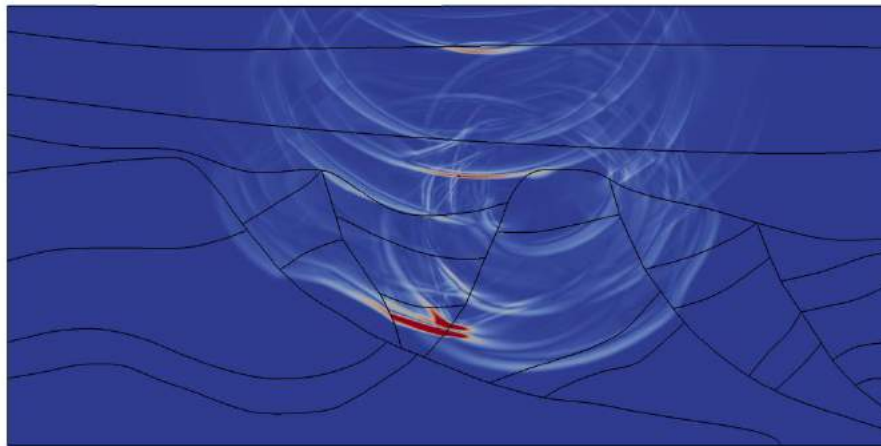
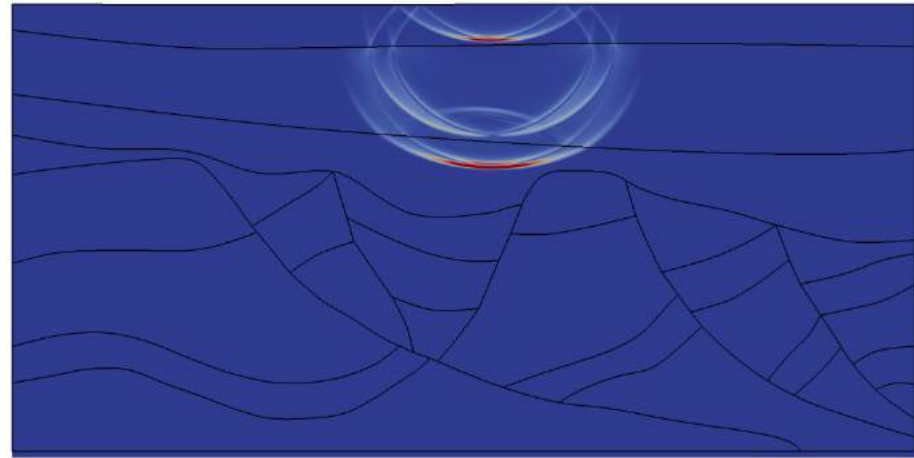
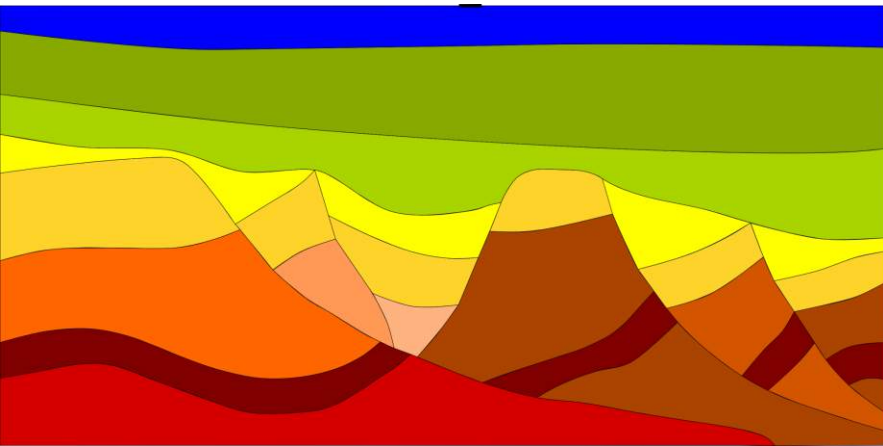


Geological model of anticlinal trap



The calculation result is the wave field at time moment  $t = 0.38$  s

# Curvilinear Borders

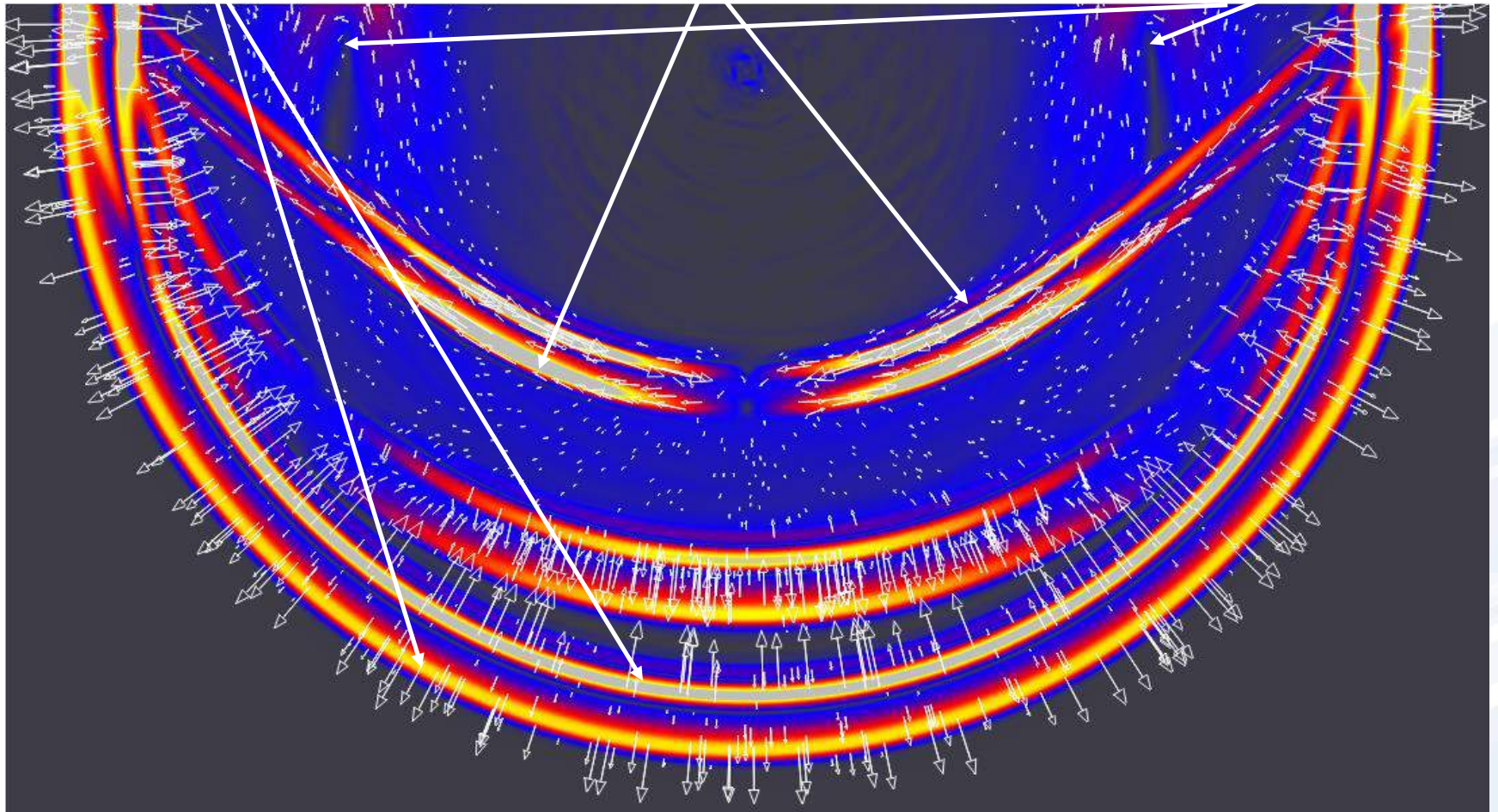


# Subsurface Explosion

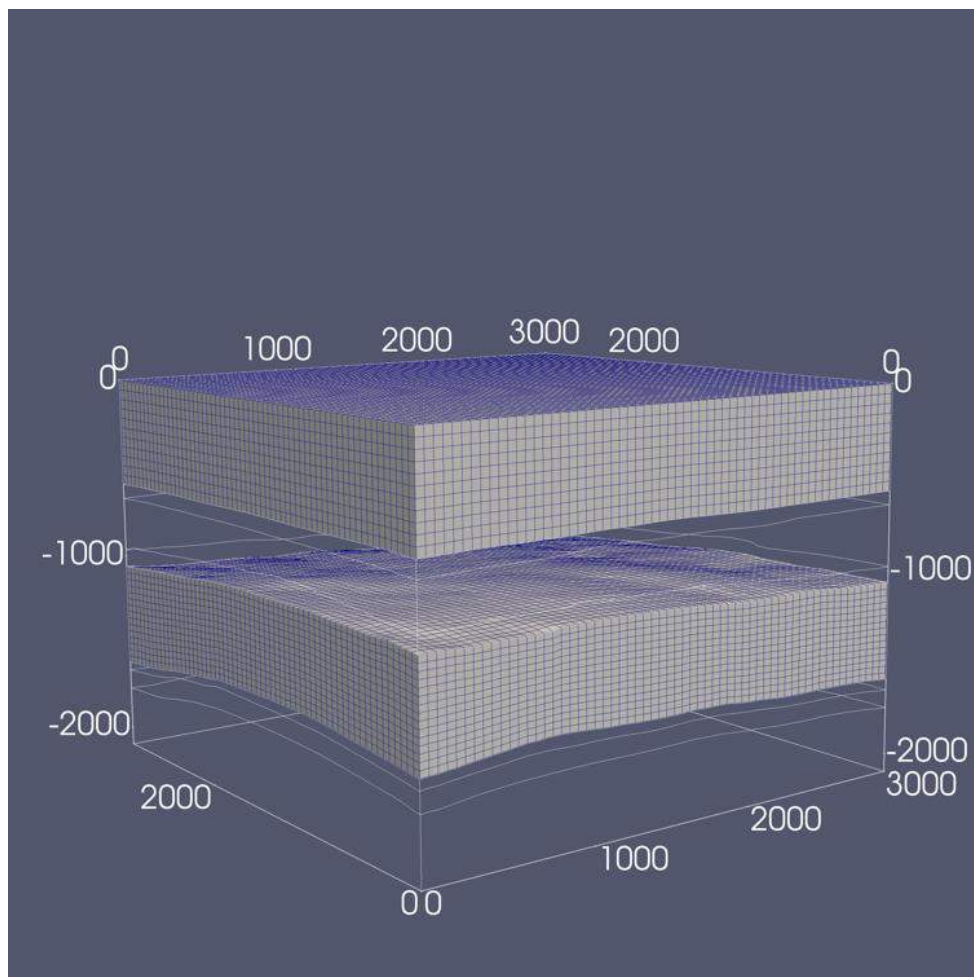
**P-waves**

**S-waves**

**Rayleigh**

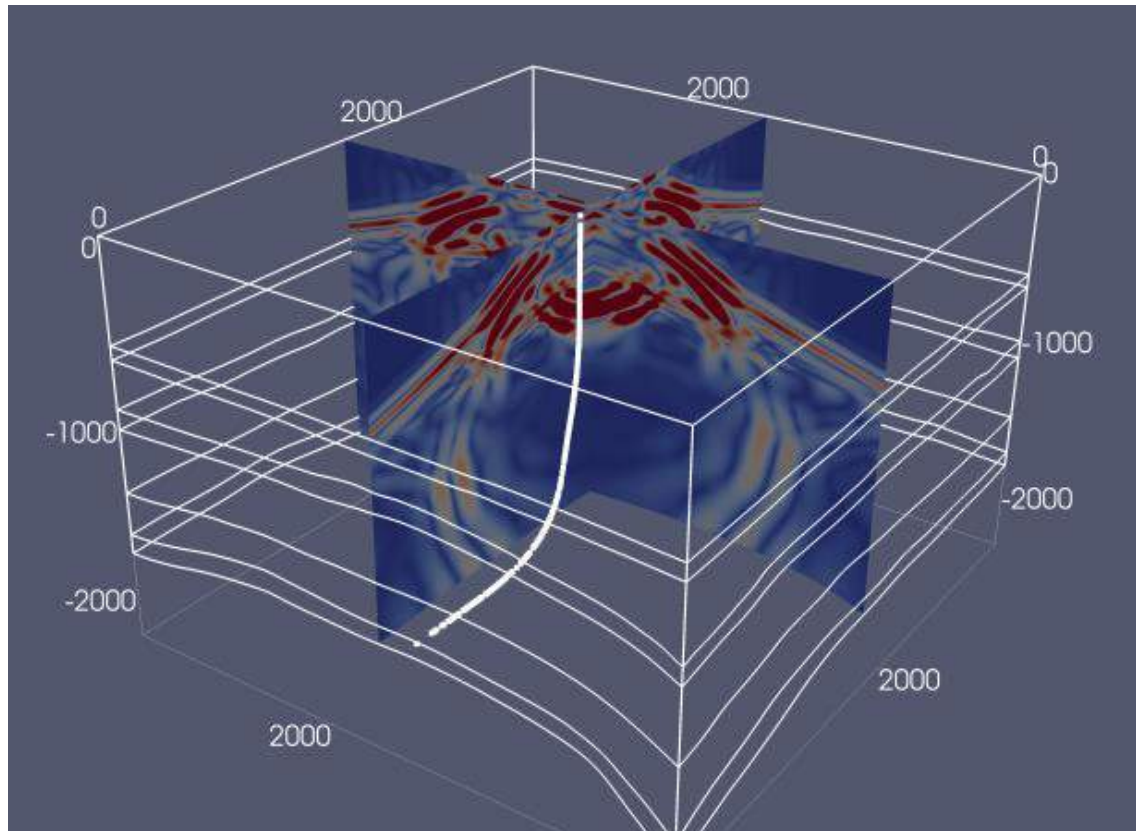


## Explicit Curvilinear Borders Benefits



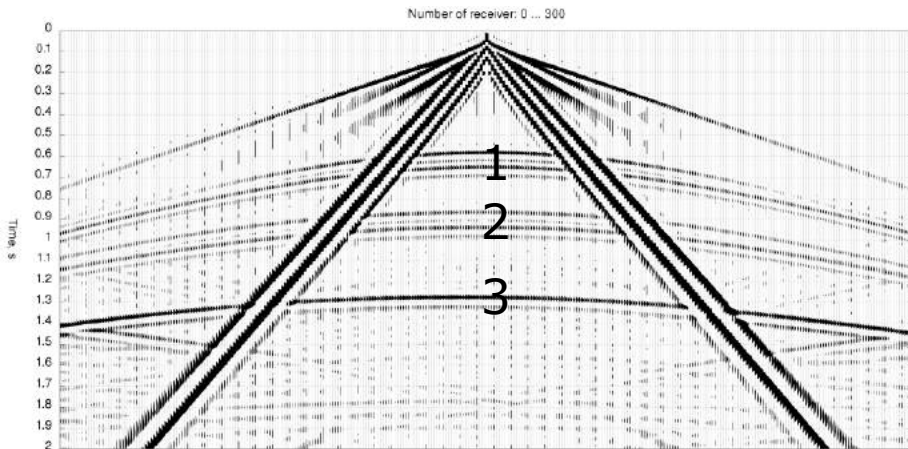
**Based on the real data**

# 3D Wave Field

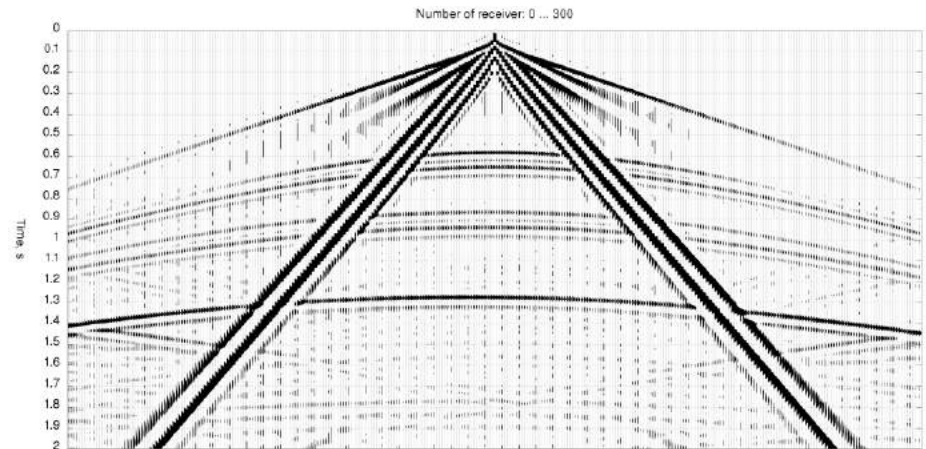


# Seismograms

## Explicit Borders Setting



## Implicit Borders Setting

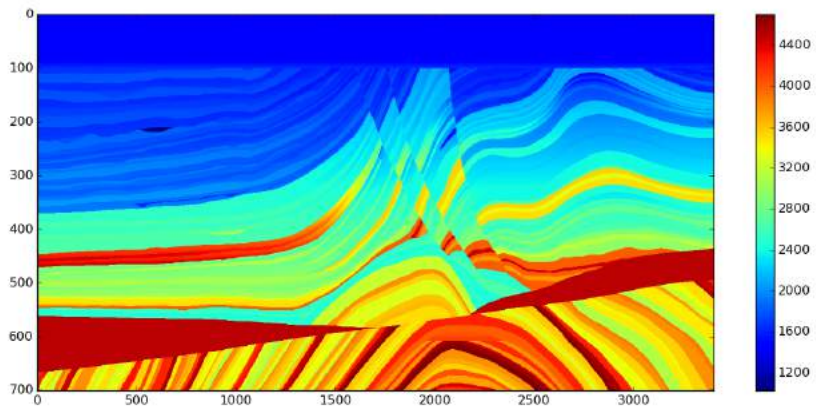


	<b>Implicit</b>	<b>Explicit</b>
Time (2 000 cores), s	815	7415
RAM, Gb	21,7	47,5
Amplitude 1		22 %
Amplitude 2		26 %
Amplitude 3		30 %

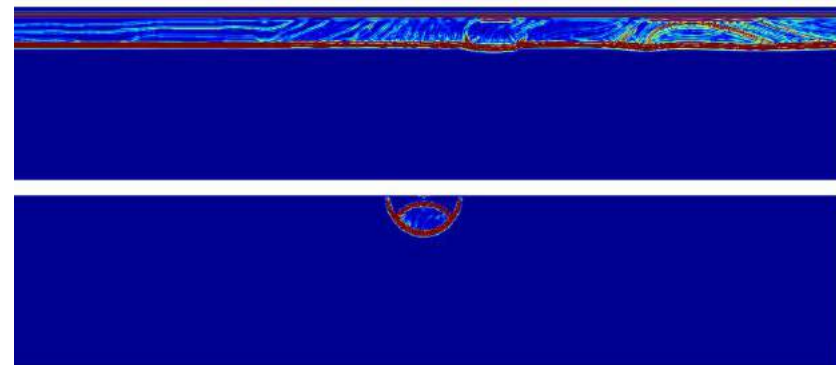
Difference up to 30 %!



# Geological model «Marmousi2»

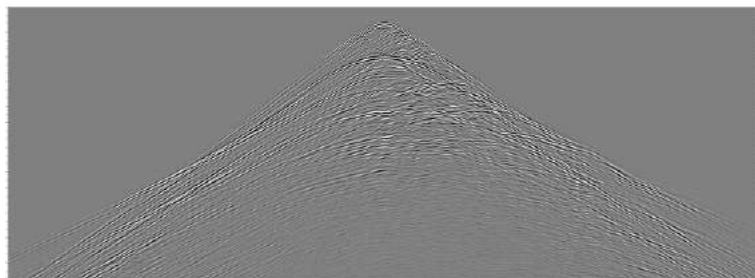
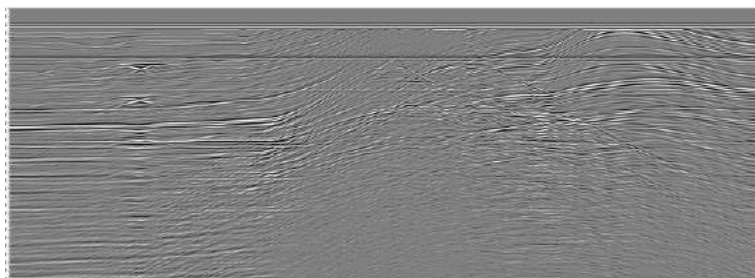
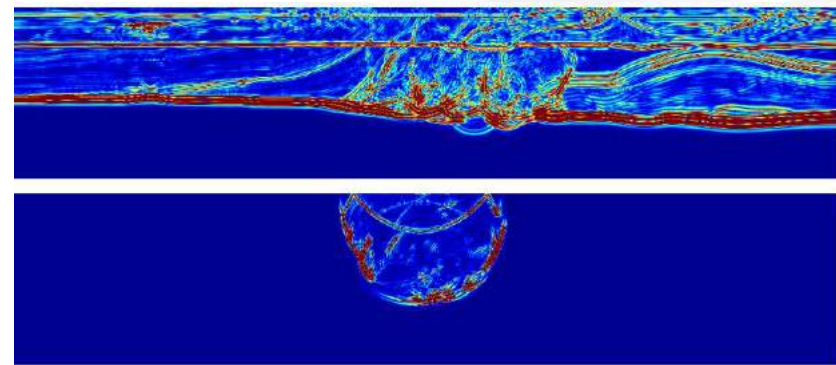


Geological model

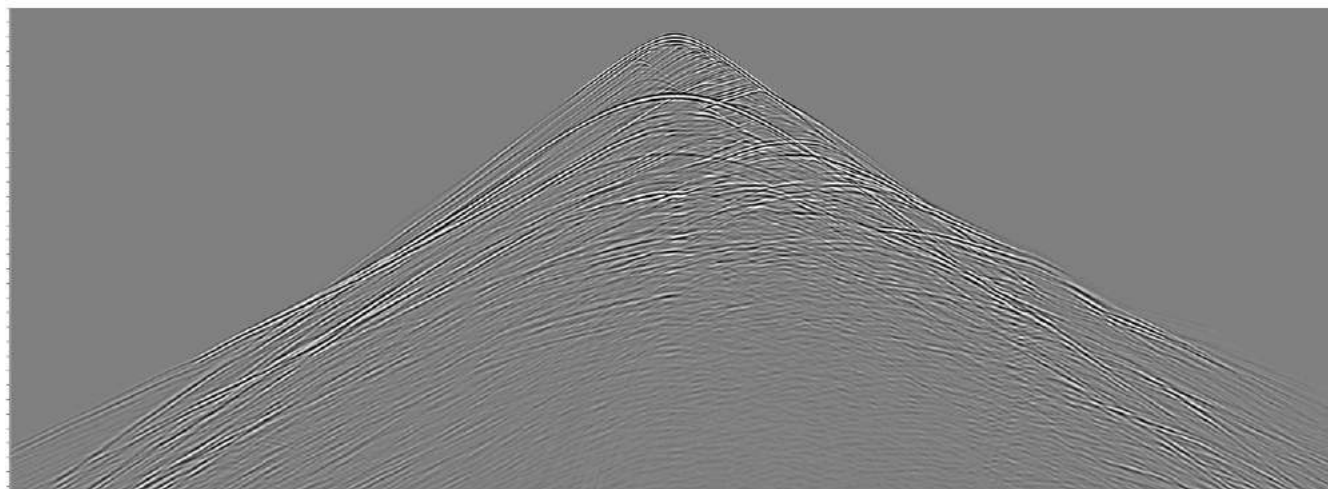
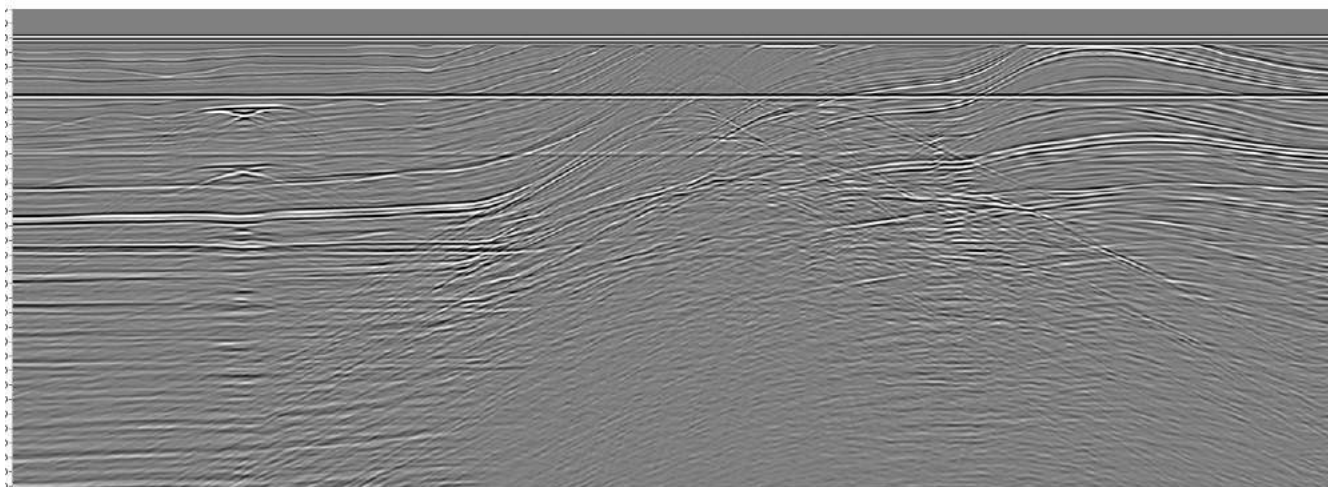


DIFFERENT TYPES OF SOURCES

- ❖ ПЛОСКАЯ ВОЛНА (СВЕРХУ)
- ❖ ТОЧЕЧНЫЙ ИСТОЧНИК (СНИЗУ)

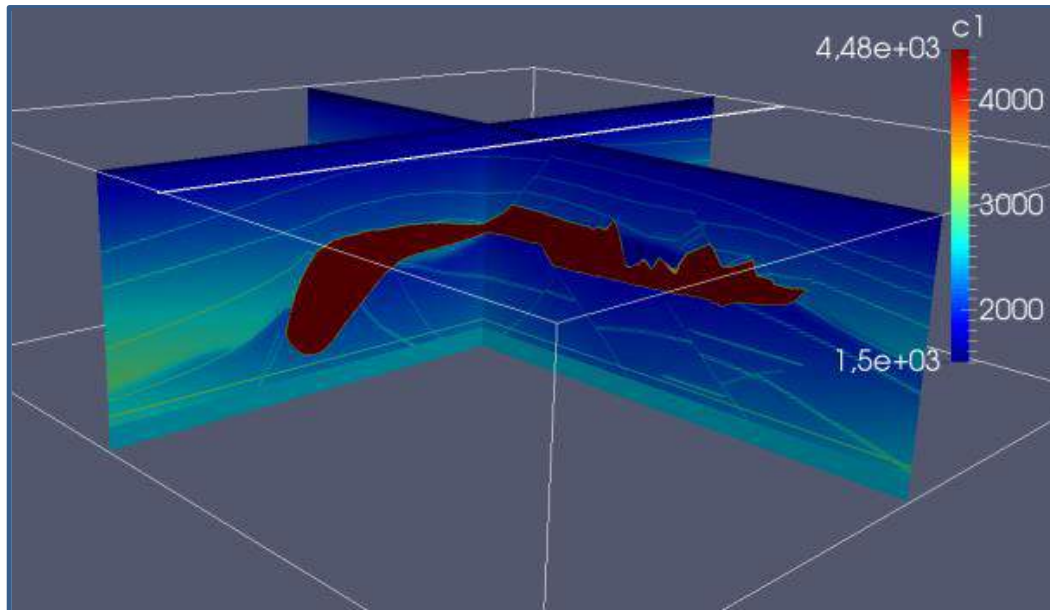


# Geological model «Marmousi2»



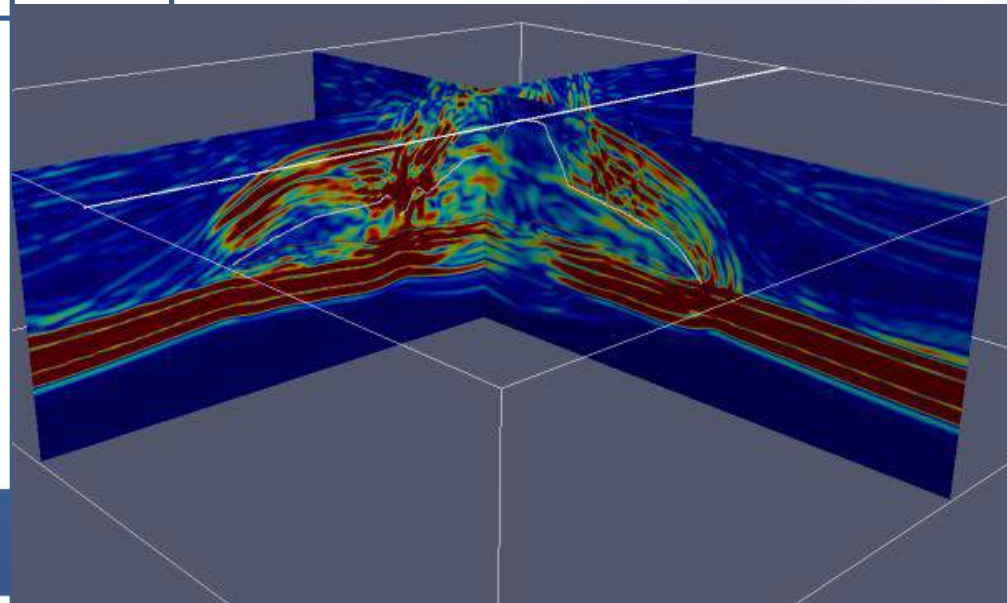
**СЕЙСМОГРАММЫ: ПЛОСКАЯ ВОЛНА (СВЕРХУ) И ТОЧЕЧНЫЙ ИСТОЧНИК (СНИЗУ)**

# «SEG C3 NA»



**3D MODEL**

## 3D WAVE FIELDS



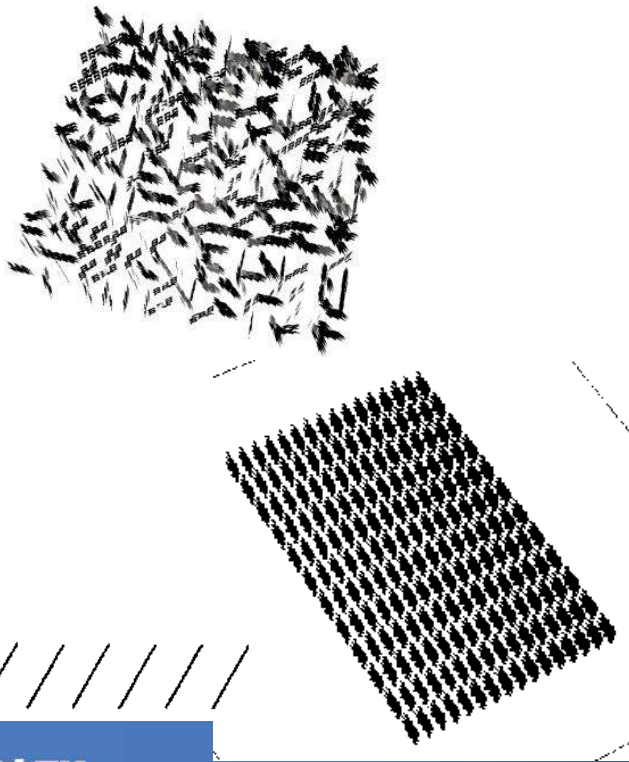
# Моделирование трещиноватых сред

Были построены следующие 3D модели:

1. однородное полупространство,
2. трёхслойная горизонтально-слоистая среда,
3. включение с вертикальными одинаково ориентированными трещинами,
4. включение с наклонными ( $30^\circ$ ) одинаково ориентированными трещинами,
5. включение с вертикальными хаотично ориентированными трещинами:
  - А. азимут от  $-30$  до  $+30$  градусов,
  - В. азимут от  $-180$  до  $+180$  градусов.

# Моделирование трещиноватых сред

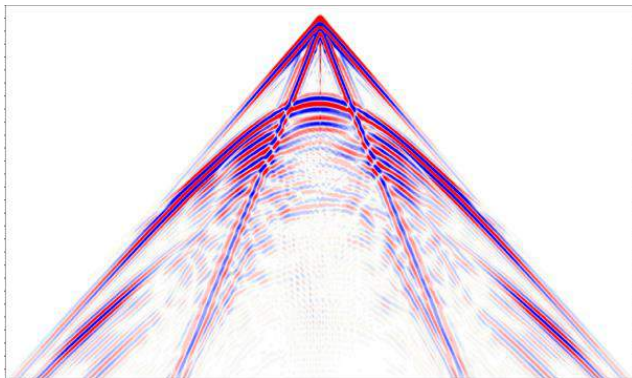
## Примеры структуры трещиноватой среды



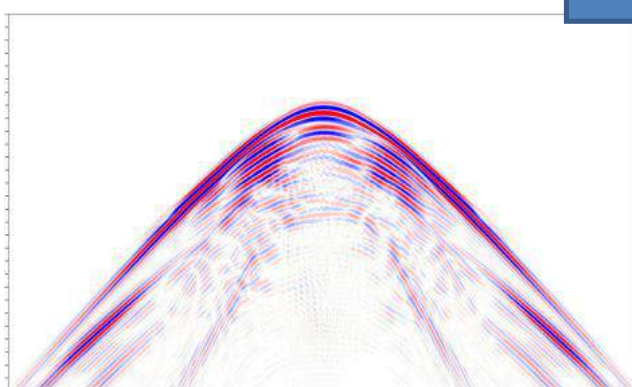
## Характеристики моделей:

- ❖ параллелепипед 10 x 10 x 3,9 км;
- ❖ плотность среды 2500 кг/м<sup>3</sup>;
- ❖ толщина 1 слоя 550 м,  $C_p$  – 4500 м/с,  $C_s$  – 2250 м/с;
- ❖ толщина 2 слоя 200 м,  $C_p$  – 6000 м/с,  $C_s$  – 3000 м/с;
- ❖ толщина 3 слоя 3150 м,  $C_p$  – 4500 м/с,  $C_s$  – 2250 м/с;
- ❖ глубина залегания трещин 600 м;
- ❖ горизонтальная протяжённость кластера 2,4 км;
- ❖ длина трещины 100 м;
- ❖ расстояние между трещинами 50-150 м.

# Методика и анализ данных ( $V_z$ )



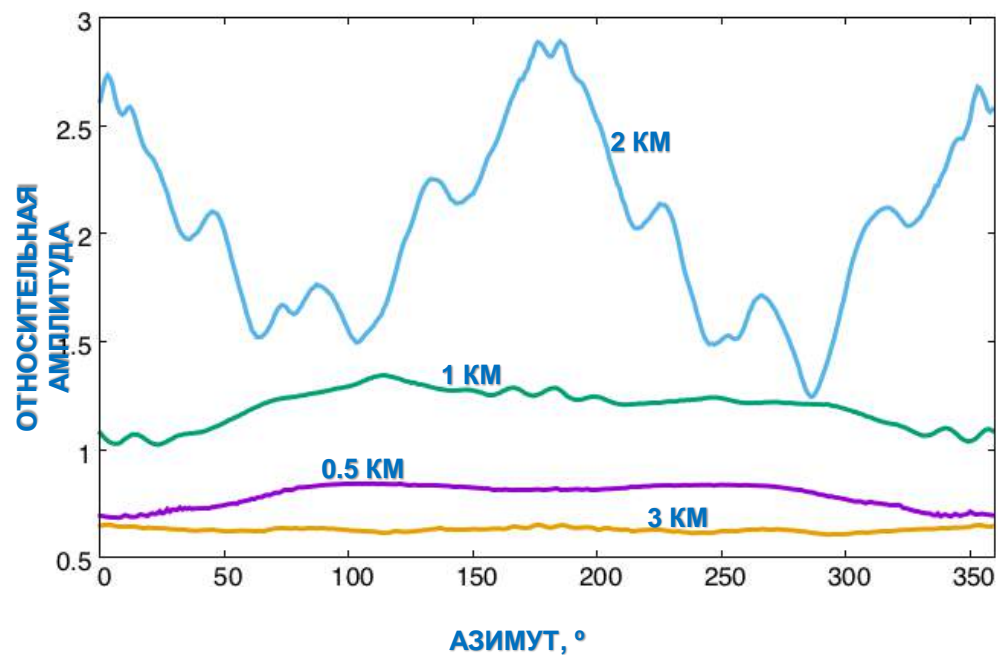
**3D ТРЕЩИНОВАТАЯ  
ГЕОЛОГИЧЕСКАЯ  
МОДЕЛЬ**



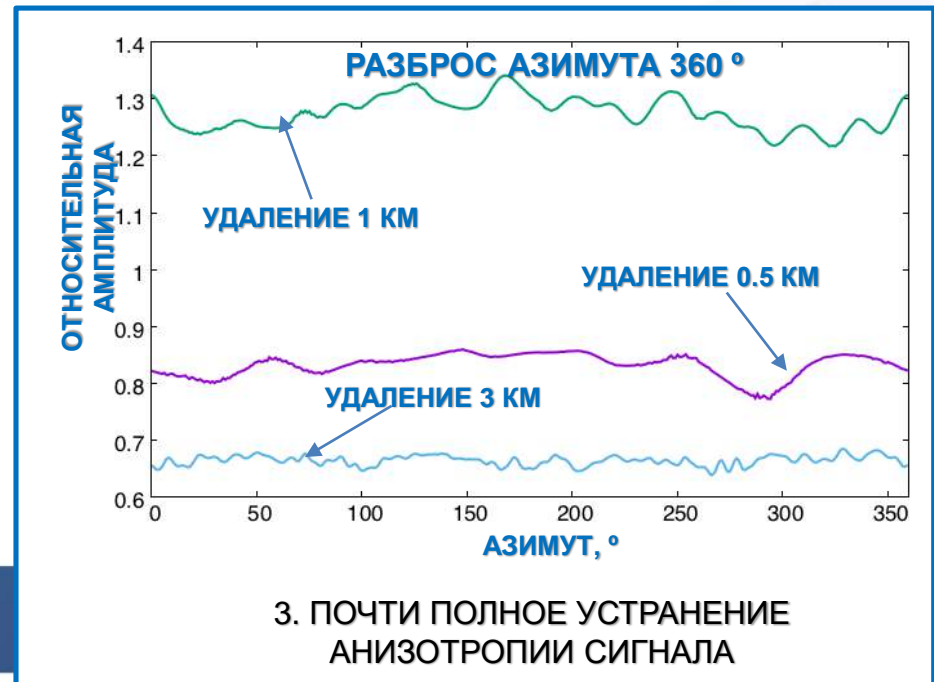
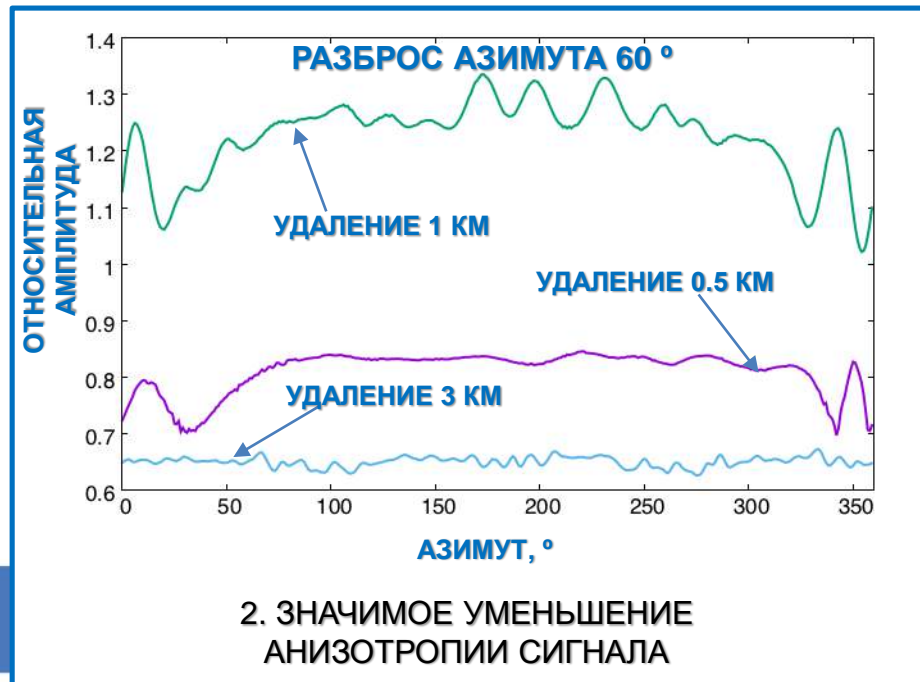
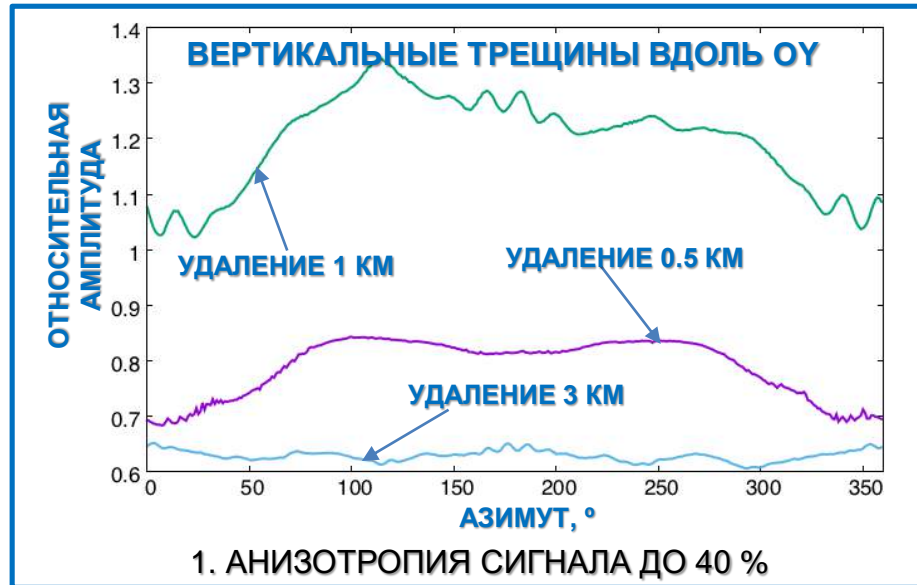
**ВЫЧТЕНА 3D  
ОДНОРОДНАЯ  
ГЕОЛОГИЧЕСКАЯ  
МОДЕЛЬ**

**Идентифицируются:**

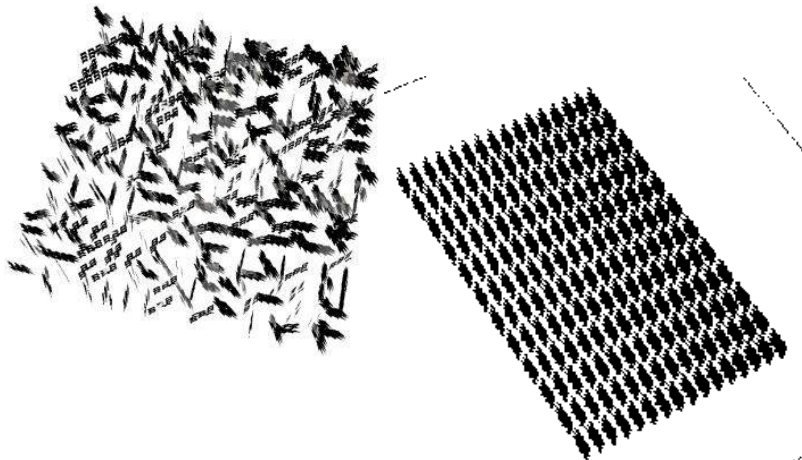
- ❖ **Поверхностные волны**
- ❖ **Отклики от границ слоёв**
- ❖ **Сейсмический сигнал от кластера трещин**



# Результаты анализа

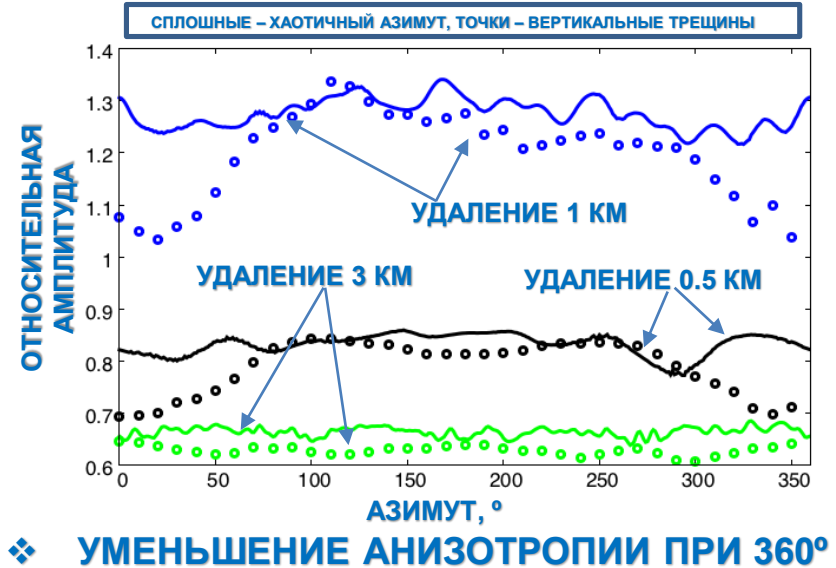
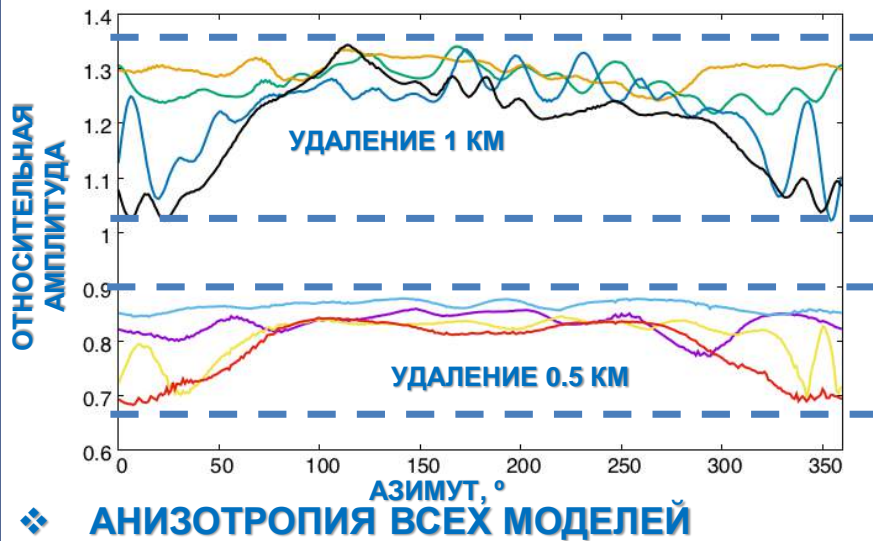


# Результаты анализа



## ГЕОМЕТРИЯ ТРЕЩИН:

- ❖ ВЕРТИКАЛЬНЫЕ
- ❖ 30° ОТ ВЕРТИКАЛИ
- ❖ РАЗБРОС ПО АЗИМУТУ 360°
- ❖ РАЗБРОС ПО АЗИМУТУ 60°





# Earthquake modeling

## **Earthquake resistance**

- Earthquake focus modeling
- Wave propagation through geological media
- Evaluation of earthquake resistance

## Problem's Significance

1. **Consequences of earthquakes** – thousands of deaths, billions \$ of damage to property
2. Available software instruments and technologies are **insufficient**



Japan

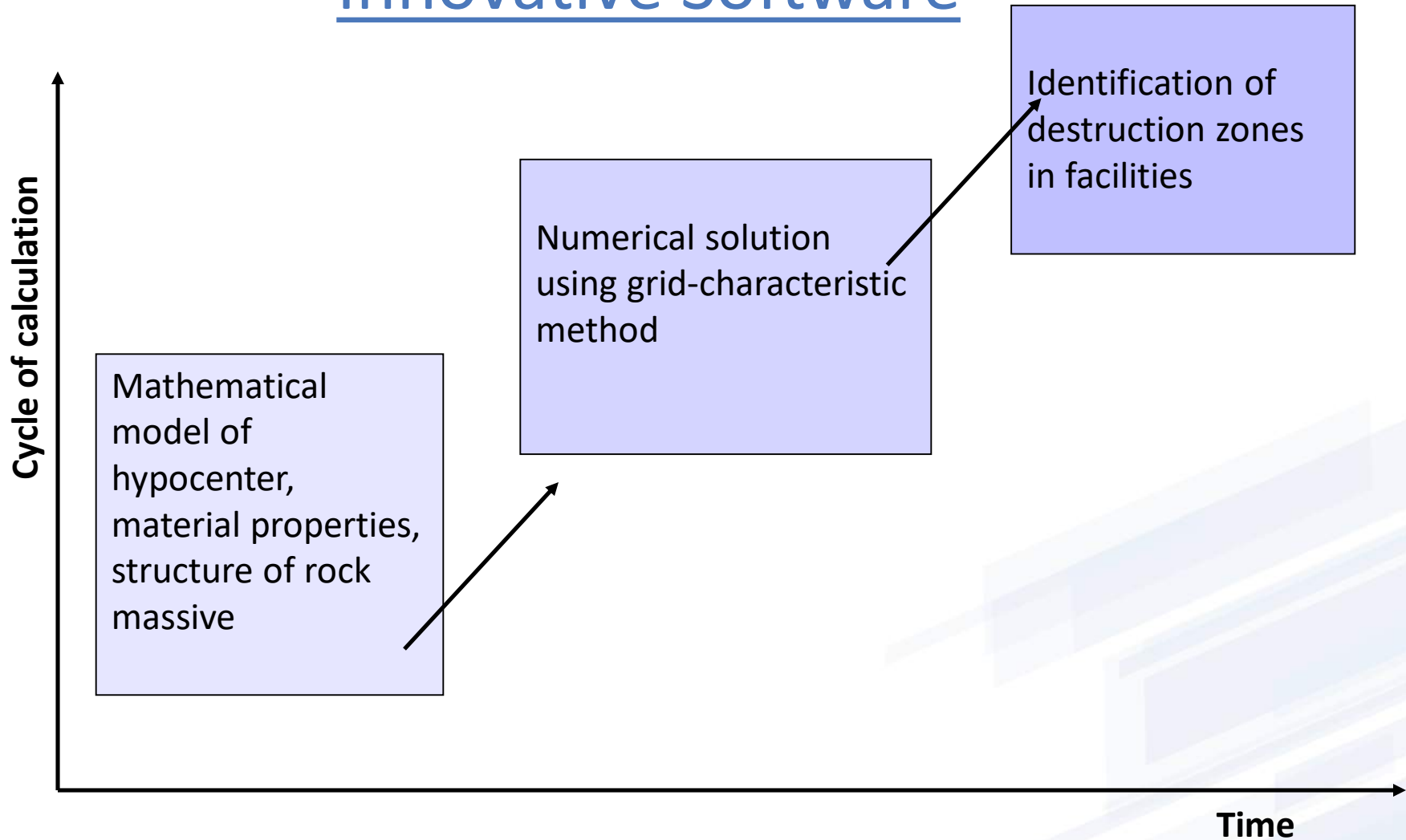


Russia



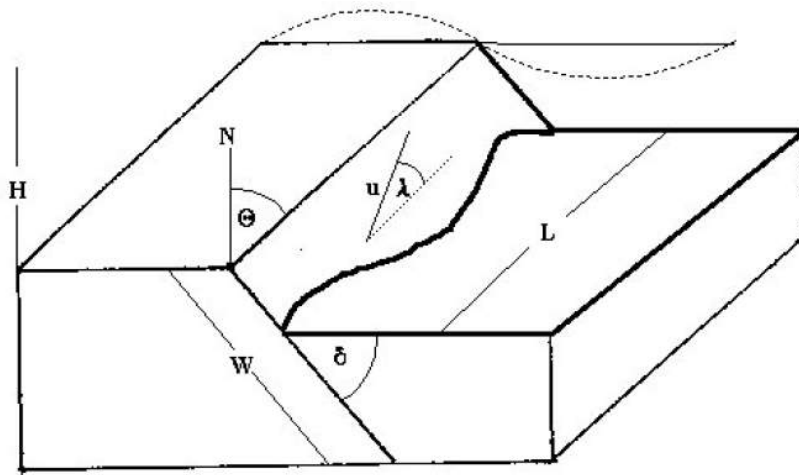
Turkey

# Innovative Software



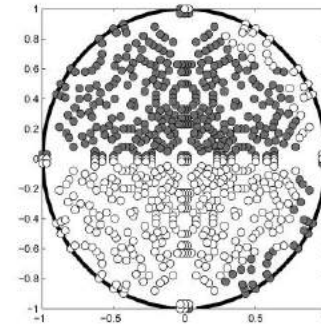
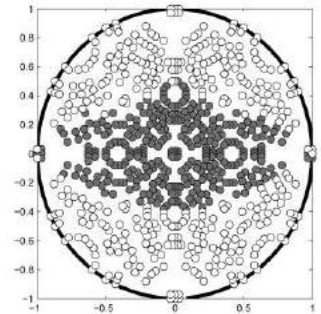
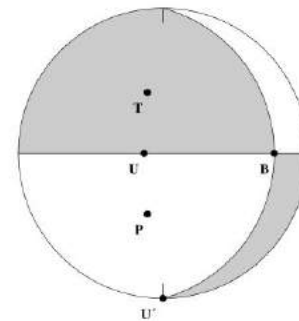
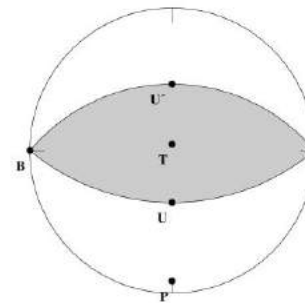
# Results: Hypocenter Modeling

Mechanical model «Fault slip»



- $\Theta$  = strike angle (measured clockwise from north)
- $\lambda$  = rake angle (angle between strike direction and slip direction;  $-\pi < \lambda \leq \pi$ )
- $\delta$  = dip angle (measured from the horizontal;  $0 \leq \delta \leq \pi/2$ )
- $u$  = slip direction and magnitude

Analytical VS numerical solution



Analytical solution is available at

<http://www1.gly.bris.ac.uk/~george/focmec.html>

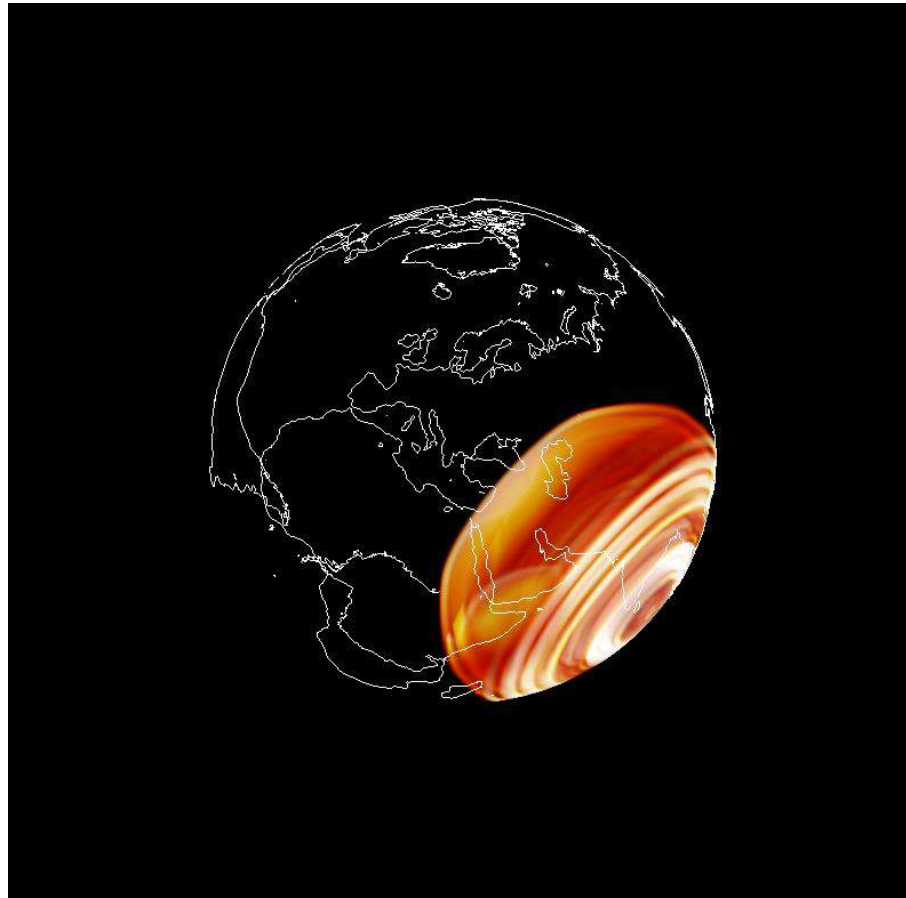
# Global seismic



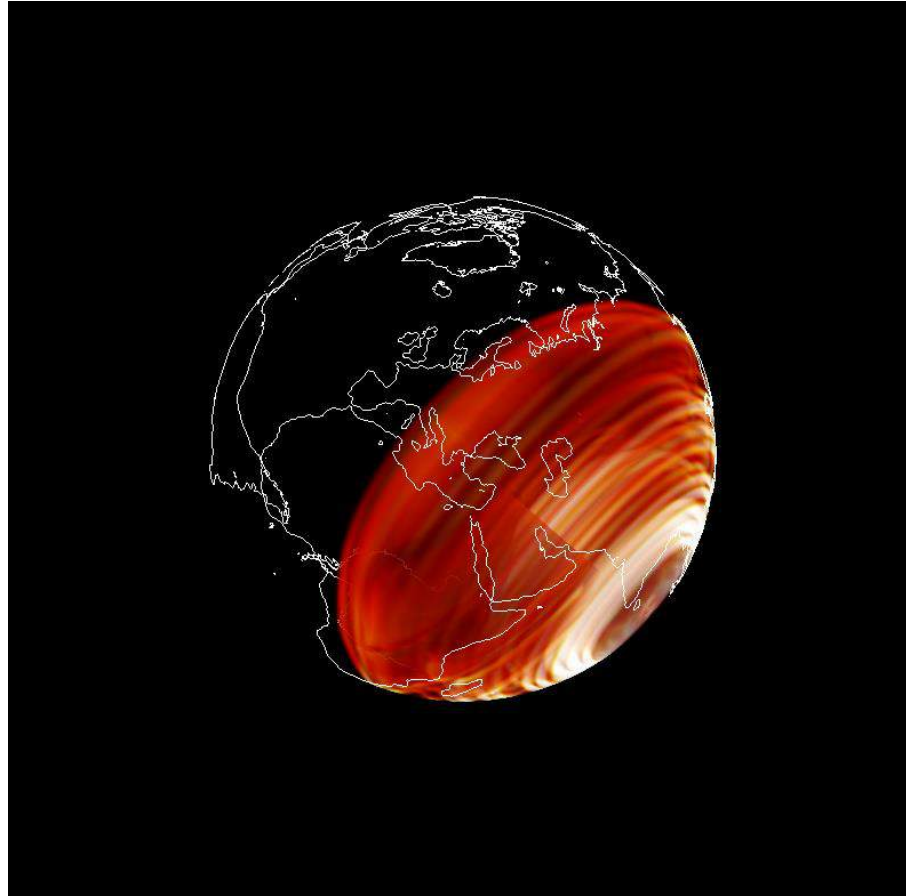
# Global seismic



# Global seismic

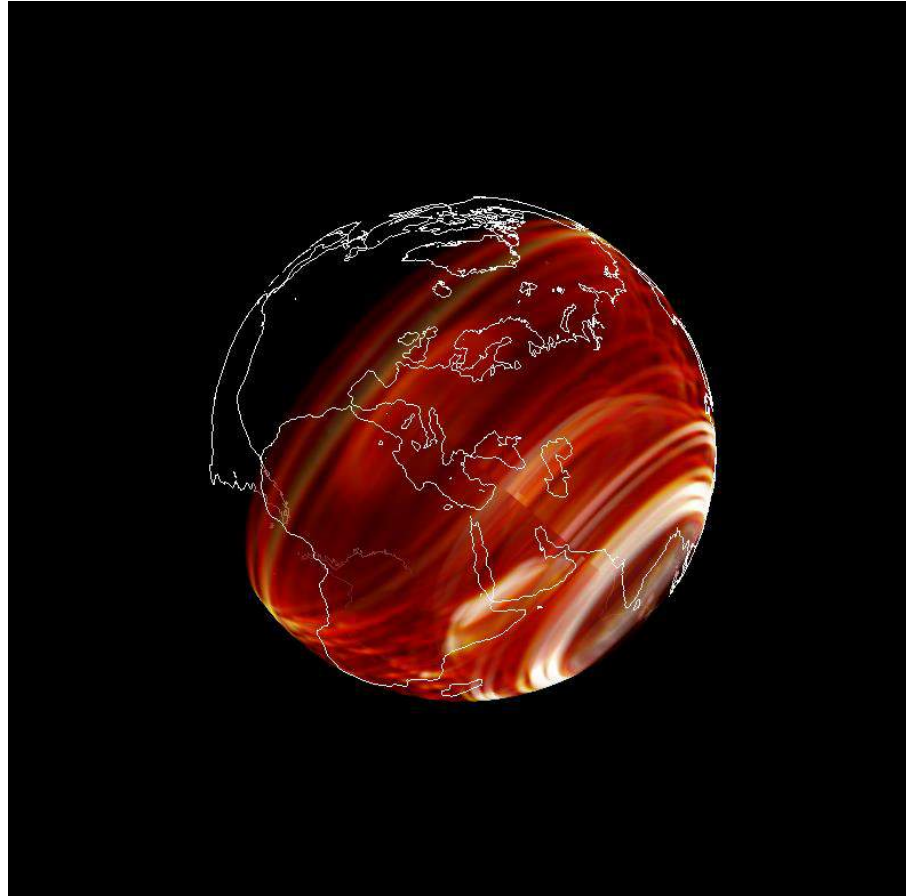


# Global seismic





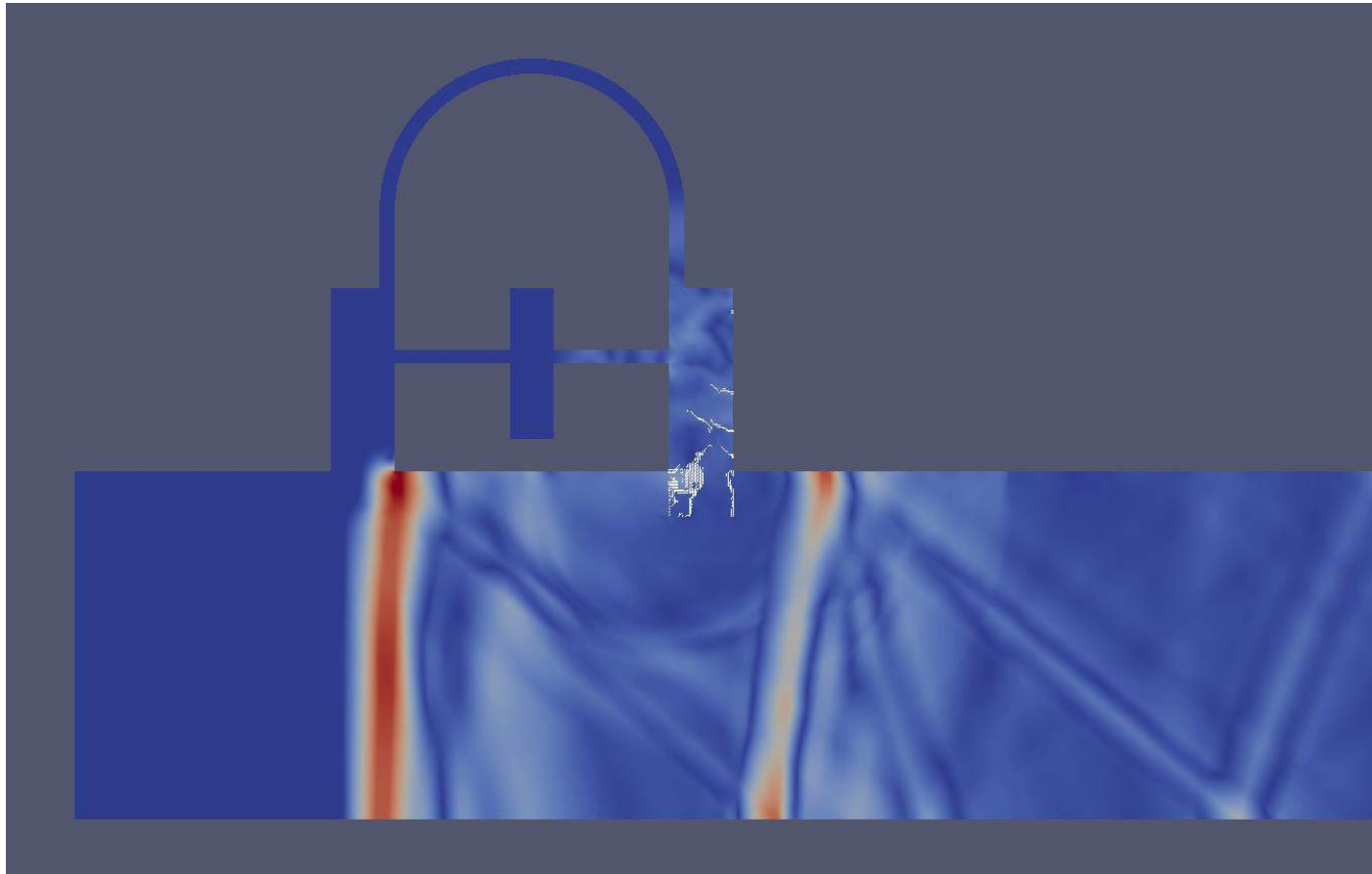
# Global seismic



# Results: dome construction



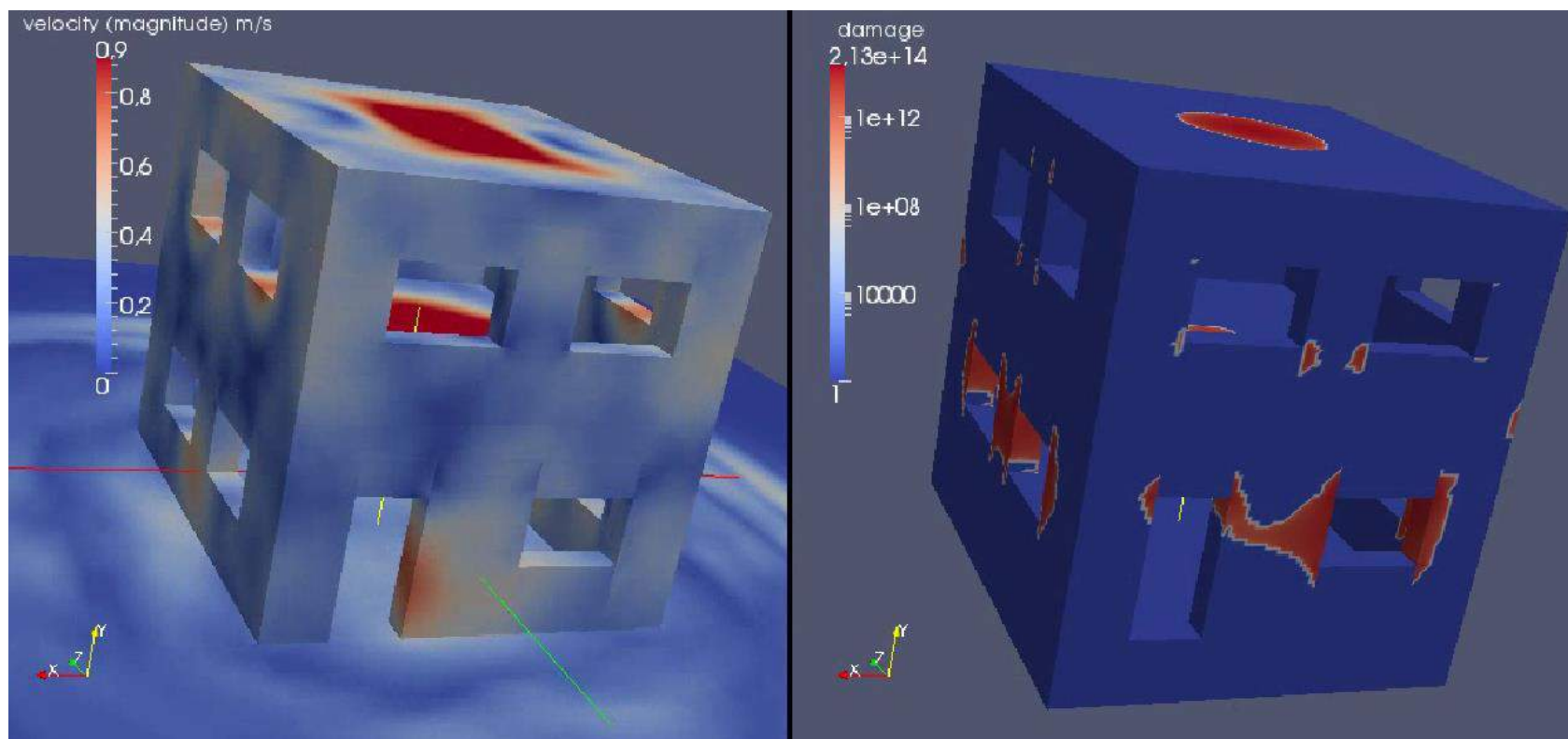
# Results: dome construction



# Results: dome construction

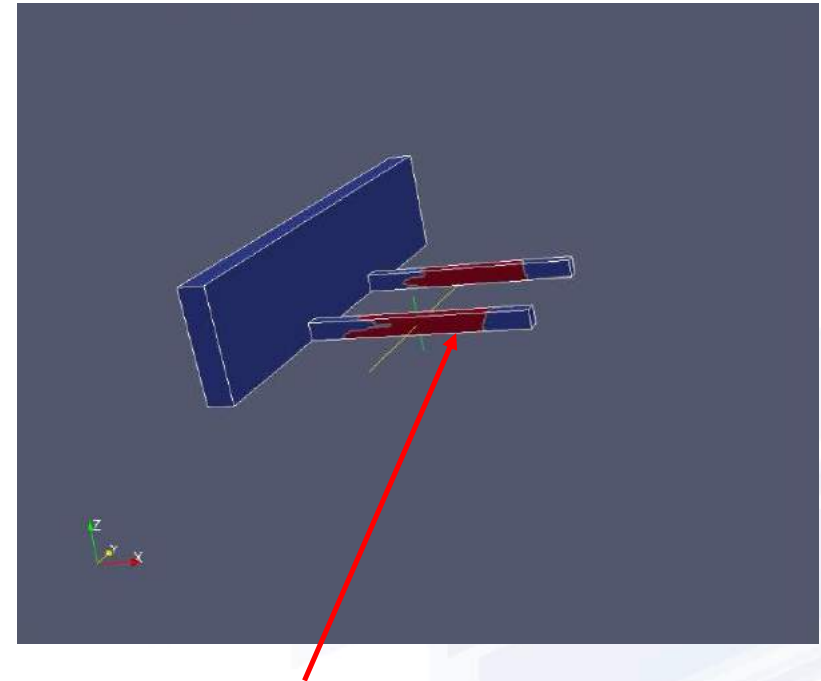
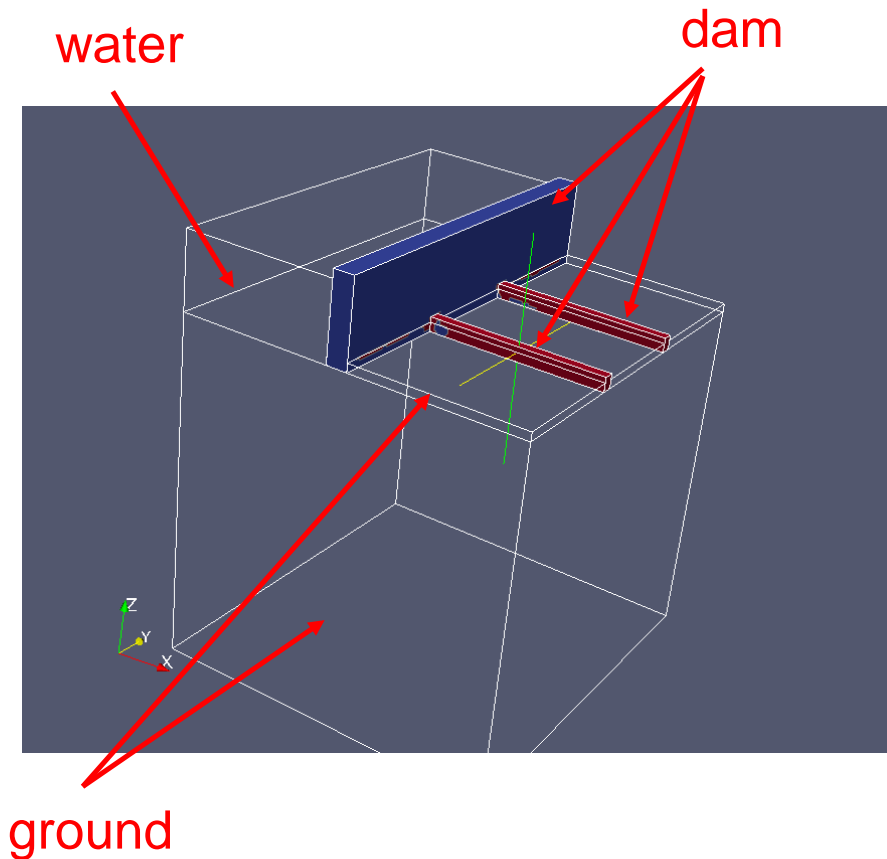


# Results: Ground Facility



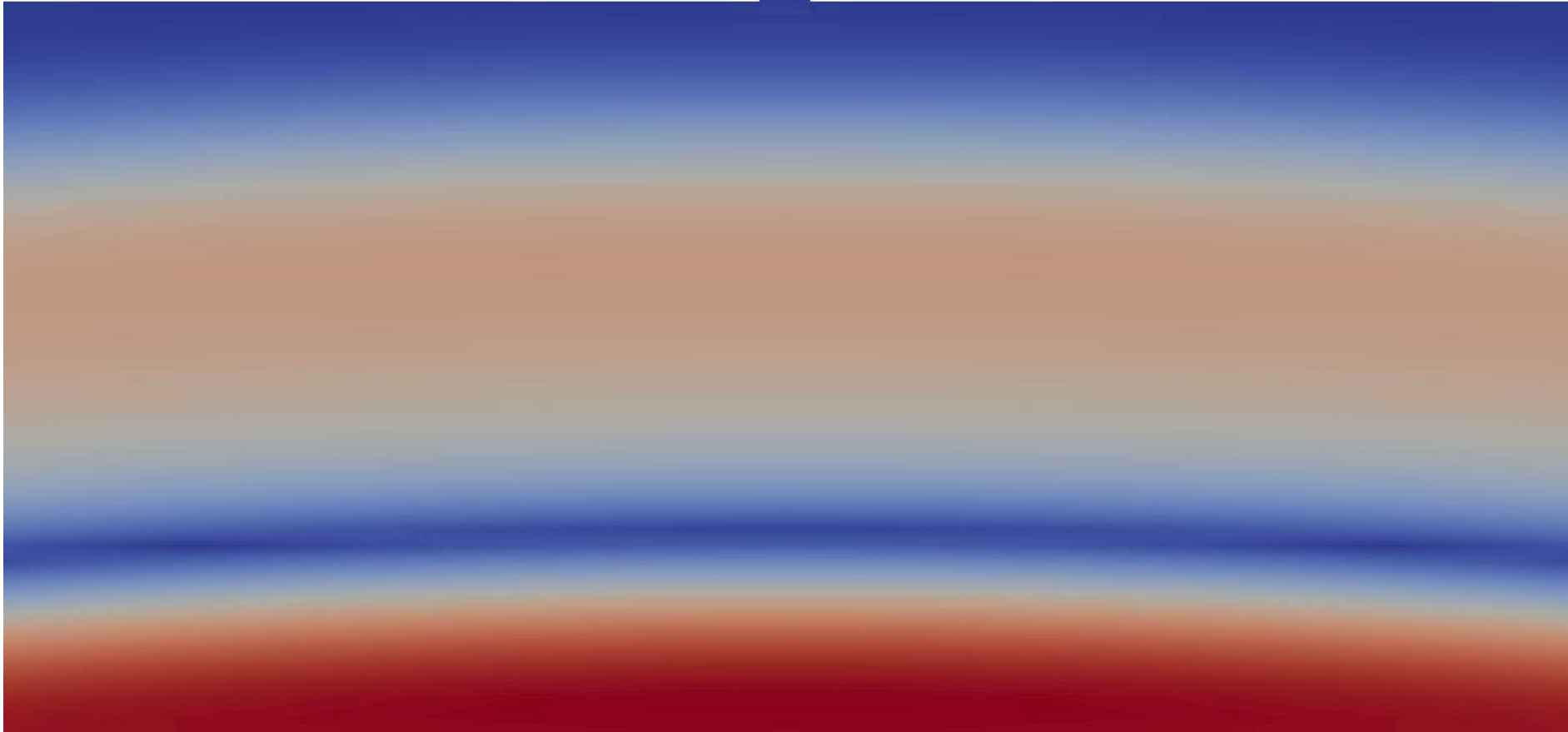
Absolute velocity (left) and destruction zones (right) in red based on «sand» model

# Results: Dam

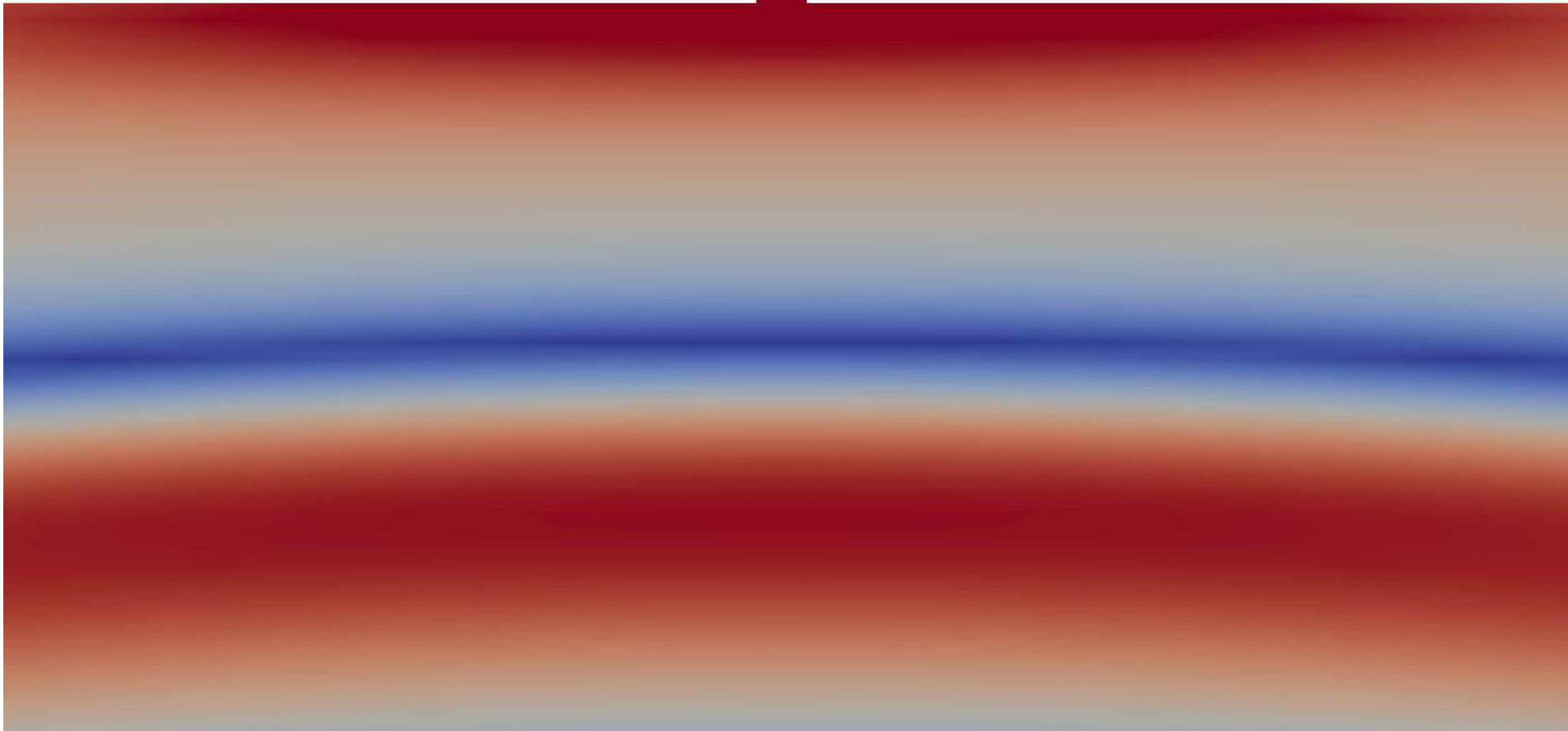


Scheme of numerical experiment and places of destruction

# Result: earthquake modelling using hierarchical grids

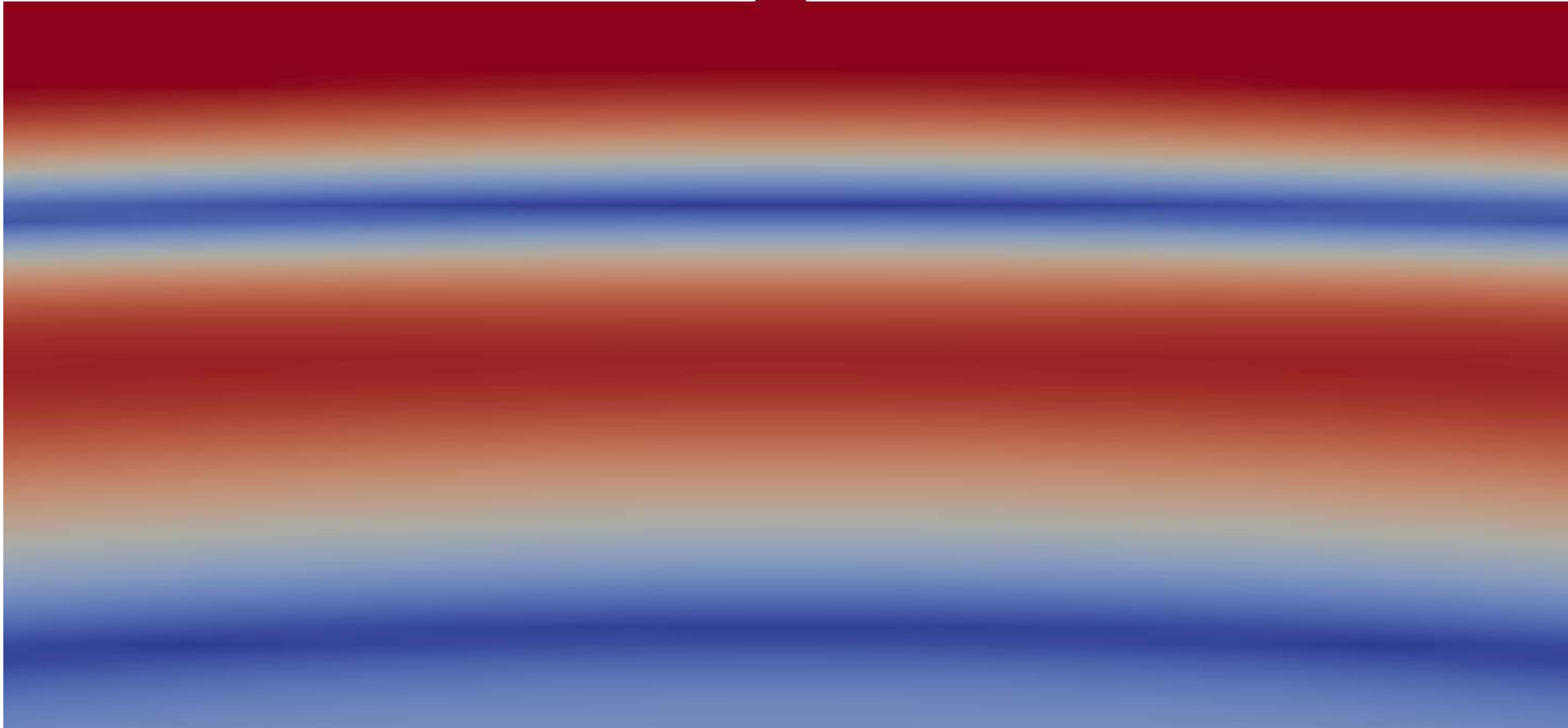


# Result: earthquake modelling using hierarchical grids

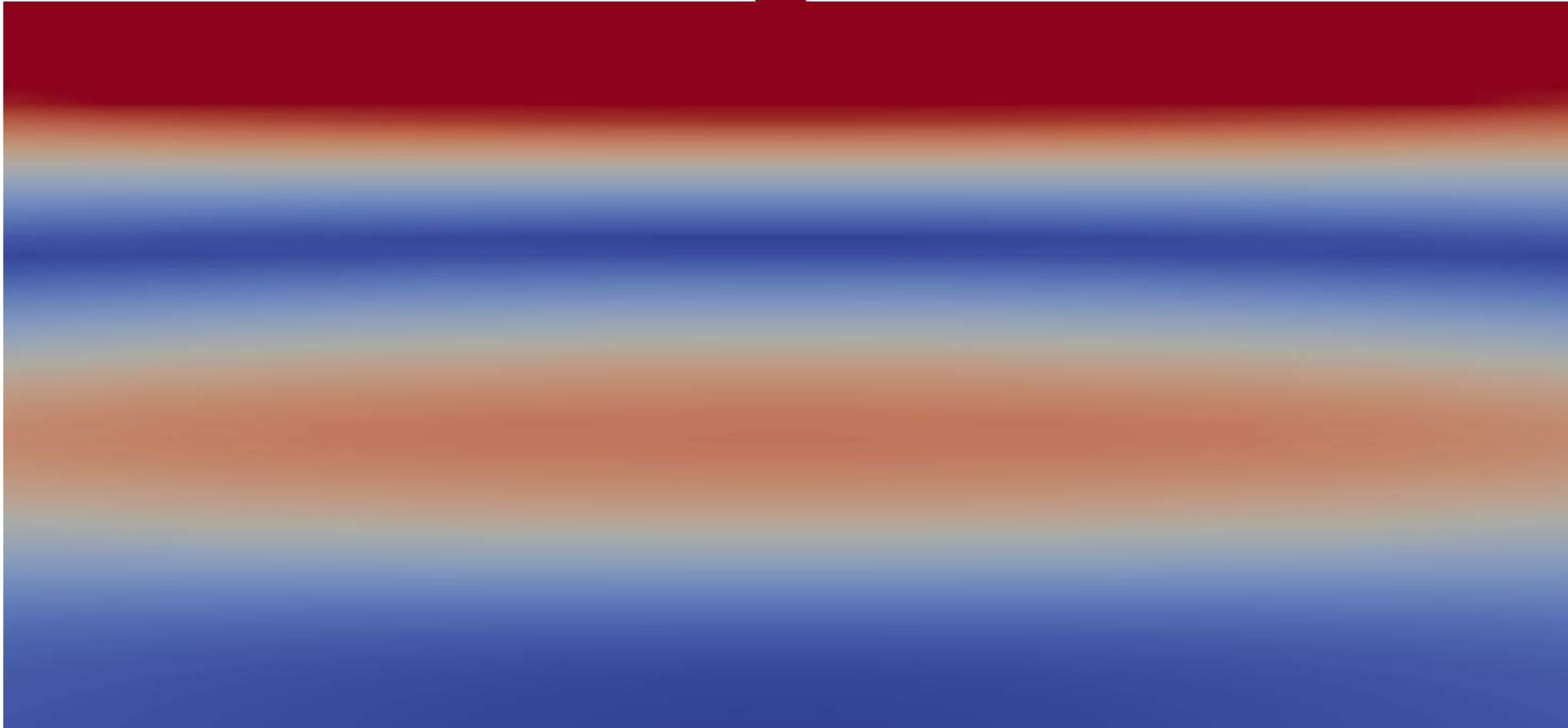




# Result: earthquake modelling using hierarchical grids



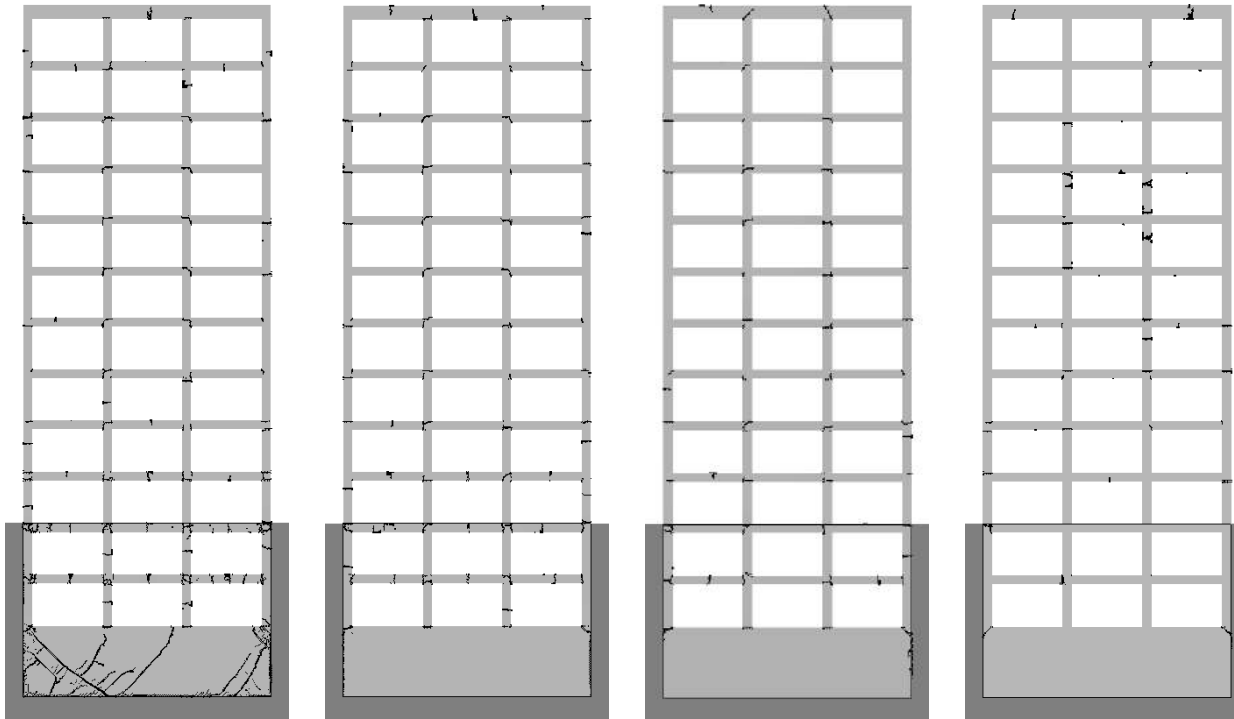
# Result: earthquake modelling using hierarchical grids



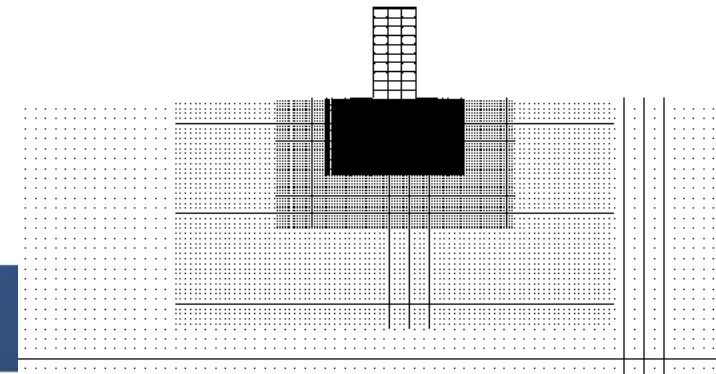
# Result: earthquake modelling using hierarchical grids



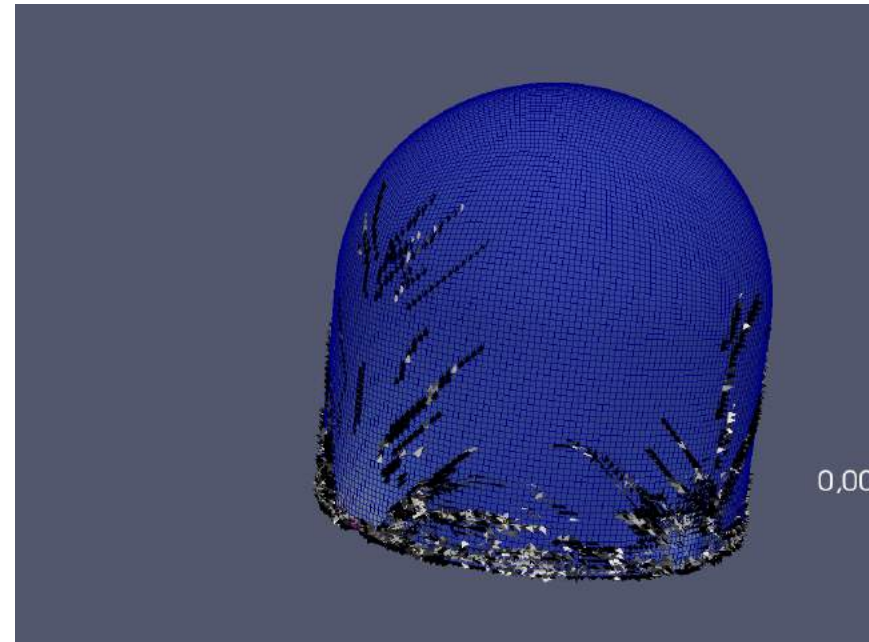
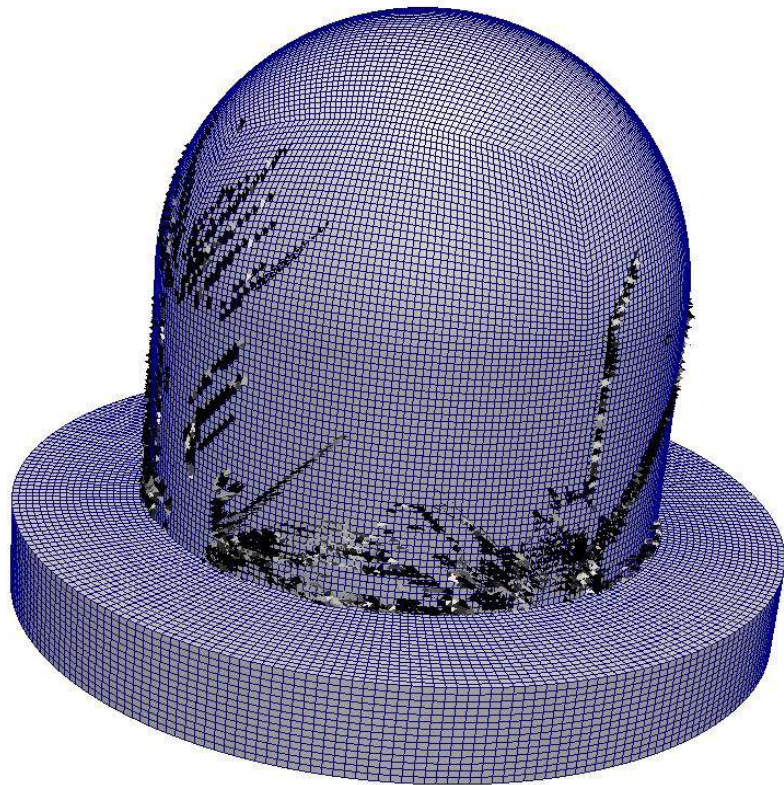
# Seismic resistance of multi-storey buildings



Different depth of the hypocenter of the earthquake



# Results: 3D dome constructions



# Conclusions

- possibility to calculate large spatial problems
- various models of destruction
- complex three-dimensional objects from block curvilinear grids

Thanks a lot!  
Questions?